SET-1

Series SSO/2

कोड नं.
Code No. **65/2/1/F**

रोल नं.
Roll No. | | | | | | | |

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **12** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **26** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **12** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **26** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 100*

Time allowed : 3 hours | *Maximum Marks : 100*

सामान्य निर्देश :

(i) **सभी** *प्रश्न अनिवार्य हैं ।*

(ii) *कृपया जाँच कर लें कि इस प्रश्न–पत्र में* **26** *प्रश्न हैं ।*

(iii) *खण्ड अ के प्रश्न* **1 – 6** *तक अति लघु-उत्तर वाले प्रश्न हैं और प्रत्येक प्रश्न के लिए* **1** *अंक निर्धारित है ।*

(iv) *खण्ड ब के प्रश्न* **7 – 19** *तक दीर्घ-उत्तर I प्रकार के प्रश्न हैं और प्रत्येक प्रश्न के लिए* **4** *अंक निर्धारित हैं ।*

(v) *खण्ड स के प्रश्न* **20 – 26** *तक दीर्घ-उत्तर II प्रकार के प्रश्न हैं और प्रत्येक प्रश्न के लिए* **6** *अंक निर्धारित हैं ।*

(vi) *उत्तर लिखना प्रारम्भ करने से पहले कृपया प्रश्न का क्रमांक अवश्य लिखिए ।*

General Instructions :

(i) ***All*** *questions are compulsory.*

(ii) *Please check that this question paper contains* ***26*** *questions.*

(iii) *Questions* ***1 – 6*** *in Section A are very short-answer type questions carrying* ***1*** *mark each.*

(iv) *Questions* ***7 – 19*** *in Section B are long-answer I type questions carrying* ***4*** *marks each.*

(v) *Questions* ***20 – 26*** *in Section C are long-answer II type questions carrying* ***6*** *marks each.*

(vi) *Please write down the serial number of the question before attempting it.*

खण्ड अ

SECTION A

प्रश्न संख्या 1 से 6 तक प्रत्येक प्रश्न का 1 अंक है ।

Question numbers 1 to 6 carry 1 mark each.

1. सदिशों $2\hat{i} + 3\hat{j} - \hat{k}$ और $4\hat{i} - 3\hat{j} + 2\hat{k}$ के योगफल के अनुदिश मात्रक सदिश ज्ञात कीजिए ।

Find the unit vector in the direction of the sum of the vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$.

2. एक समान्तर चतुर्भज का क्षेत्रफल ज्ञात कीजिए जिसकी संलग्न भुजाएँ सदिशों $2\hat{i} - 3\hat{k}$ तथा $4\hat{j} + 2\hat{k}$ द्वारा निर्धारित हैं ।

Find the area of a parallelogram whose adjacent sides are represented by the vectors $2\hat{i} - 3\hat{k}$ and $4\hat{j} + 2\hat{k}$.

3. समतल $2x + y - z = 5$ द्वारा निर्देशांक अक्षों पर काटे गए अंतःखण्डों का योगफल ज्ञात कीजिए :

Find the sum of the intercepts cut off by the plane $2x + y - z = 5$, on the coordinate axes.

4. यदि $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, तो दूसरी पंक्ति के अवयव a_{21} का सहखण्ड लिखिए ।

If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then write the cofactor of the element a_{21} of its 2^{nd} row.

5. अवकल समीकरण $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ की कोटि व घात का योगफल लिखिए ।

Write the sum of the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0.$$

6. अवकल समीकरण $\frac{dy}{dx} = 2^{-y}$ का हल लिखिए ।

Write the solution of the differential equation

$$\frac{dy}{dx} = 2^{-y}.$$

खण्ड ब

SECTION B

प्रश्न संख्या 7 से 19 तक प्रत्येक प्रश्न के 4 अंक हैं ।

Question numbers 7 to 19 carry 4 marks each.

7. यदि $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ और I एक 2 कोटि का तत्समक आव्यूह हो, तो दिखाइए कि $A^2 = 4A - 3I$. अत: A^{-1} ज्ञात कीजिए ।

अथवा

यदि $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ और $(A + B)^2 = A^2 + B^2$ है, तो a और b के मान ज्ञात कीजिए ।

If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1}.

OR

If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then find the values of a and b.

8. सारणिकों के गुणधर्मों का प्रयोग करके, निम्नलिखित को सिद्ध कीजिए :

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = \left(1 - a^3\right)^2$$

Using properties of determinants, prove the following :

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = \left(1 - a^3\right)^2$$

9. मान ज्ञात कीजिए :

$$\int \frac{\sin(x-a)}{\sin(x+a)}\, dx$$

अथवा

मान ज्ञात कीजिए :

$$\int \frac{x^2}{(x^2+4)(x^2+9)}\, dx$$

Evaluate :

$$\int \frac{\sin(x-a)}{\sin(x+a)}\, dx$$

OR

Evaluate :

$$\int \frac{x^2}{(x^2+4)(x^2+9)}\, dx$$

10. मान ज्ञात कीजिए :

$$\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}\, dx$$

Evaluate :

$$\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}\, dx$$

11. बिजली के बल्ब बनाने वाली एक कम्पनी में तीन मशीनें E_1, E_2 और E_3 दिनभर के उत्पादन का क्रमशः 50%, 25% तथा 25% भाग तैयार करती हैं । यह ज्ञात है कि मशीन E_1 और E_2 प्रत्येक से बने बल्बों में 4% खराब बल्ब होते हैं, और मशीन E_3 से बने बल्बों में 5% खराब होते हैं । यदि दिन के उत्पादन में से एक बल्ब यादृच्छया चुना जाए, तो इस बल्ब के खराब होने की प्रायिकता ज्ञात कीजिए ।

अथवा

धन पूर्णांकों 2, 3, 4, 5, 6 तथा 7 में से दो संख्याएँ यादृच्छया (बिना प्रतिस्थापन) चुनी गईं । मान लीजिए X दोनों संख्याओं में से बड़ी संख्या को व्यक्त करता है । X के प्रायिकता बंटन का माध्य तथा प्रसरण ज्ञात कीजिए ।

Three machines E_1, E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective.

OR

Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6, and 7. Let X denote the larger of the two numbers obtained. Find the mean and variance of the probability distribution of X.

12. दो सदिश $\hat{j} + \hat{k}$ तथा $3\hat{i} - \hat{j} + 4\hat{k}$ त्रिभुज ABC के दो भुजा सदिश क्रमशः $\overrightarrow{AB}$ तथा $\overrightarrow{AC}$ को निरूपित करते हैं । A से गुज़रने वाली माध्यिका की लंबाई ज्ञात कीजिए ।

The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC. Find the length of the median through A.

13. उस समतल का समीकरण ज्ञात कीजिए, जो बिन्दु (3, 2, 0) से गुज़रता हो तथा रेखा $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ को अन्तर्विष्ट करता हो ।

Find the equation of a plane which passes through the point (3, 2, 0) and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

14. यदि $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\operatorname{cosec}\theta)$, $(\theta \neq 0)$, तो θ का मान ज्ञात कीजिए ।

अथवा

यदि $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1+n.(n+1)}\right) = \tan^{-1}\theta$ है, तो θ का मान ज्ञात कीजिए ।

If $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\operatorname{cosec}\theta)$, $(\theta \neq 0)$, then find the value of θ.

OR

If $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1+n.(n+1)}\right) = \tan^{-1}\theta$, then find the value of θ.

15. वक्र $9y^2 = x^3$ का वह बिन्दु ज्ञात कीजिए जिस पर वक्र पर खींचा गया अभिलम्ब अक्षों पर एकसमान अंत:खण्ड काटता हो ।

Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts on the axes.

16. यदि $y = \left(x + \sqrt{1+x^2}\right)^n$ है, तो दिखाइए कि

$$\left(1+x^2\right) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = n^2y.$$

If $y = \left(x + \sqrt{1+x^2}\right)^n$, then show that

$$\left(1+x^2\right) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = n^2y.$$

17. ज्ञात कीजिए कि निम्नलिखित फलन x = 1 तथा x = 2 पर अवकलनीय है अथवा नहीं :

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

Find whether the following function is differentiable at x = 1 and x = 2 or not :

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

18. संसदीय चुनाव में, एक राजनीतिक पार्टी ने प्रचार करने वाली एक फर्म को पार्टी के उम्मीदवारों को प्रचार करने में सहयोग देने के लिए नियुक्त किया । प्रचार तीन तरीकों से करना था — टेलीफोन से, घर-घर जाकर मिलना तथा पत्र-व्यवहार से । प्रति यूनिट (सम्पर्क) का खर्चा (पैसों में) निम्न आव्यूह A से नीचे दिया गया है :

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \begin{array}{l} \text{टेलीफोन} \\ \text{घर-घर जाकर मिलना} \\ \text{पत्र-व्यवहार} \end{array}$$

दो शहरों X तथा Y में प्रत्येक प्रकार के कुल यूनिटों (सम्पर्कों) का विवरण नीचे आव्यूह B में दिया गया है :

$$\begin{array}{c} \\ B = \end{array} \begin{array}{c} \begin{array}{ccc} \text{टेलीफोन} & \text{घर-घर जाकर मिलना} & \text{पत्र-व्यवहार} \end{array} \\ \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{शहर X} \\ \text{शहर Y} \end{array}$$

पार्टी ने दोनों शहरों में कुल कितना खर्च किया ?

आपके ख्याल में आप अपना वोट देने से पहले पार्टी की किस प्रकार की गतिविधि को ज़यादा महत्त्व देंगे — प्रचार गतिविधि या उनकी सामाजिक गतिविधियाँ ?

In a parliament election, a political party hired a public relations firm to promote its candidates in three ways — telephone, house calls and letters. The cost per contact (in paise) is given in matrix A as

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \begin{array}{l} \text{Telephone} \\ \text{House Call} \\ \text{Letters} \end{array}$$

The number of contacts of each type made in two cities X and Y is given in the matrix B as

$$\begin{array}{c} \\ B = \end{array} \begin{array}{c} \begin{array}{ccc} \text{Telephone} & \text{House Call} & \text{Letters} \end{array} \\ \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{City X} \\ \text{City Y} \end{array}$$

Find the total amount spent by the party in the two cities.

What should one consider before casting his/her vote — party's promotional activity or their social activities ?

19. मान ज्ञात कीजिए :

$$\int e^{2x} . \sin(3x+1)\, dx$$

Evaluate :

$$\int e^{2x} . \sin(3x+1)\, dx$$

खण्ड स
SECTION C

प्रश्न संख्या 20 से 26 तक प्रत्येक प्रश्न के 6 अंक हैं ।
Question numbers 20 to 26 carry 6 marks each.

20. माना $f : N \to \mathbb{R}$, $f(x) = 4x^2 + 12x + 15$ द्वारा परिभाषित एक फलन है । सिद्ध कीजिए कि $f : N \to S$, जहाँ S, फलन f का परिसर है, व्युत्क्रमणीय है । f का प्रतिलोम भी ज्ञात कीजिए ।

Let $f : N \to \mathbb{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where S is the range of f, is invertible. Also find the inverse of f.

21. समाकलन विधि से रेखा $x - y + 2 = 0$, वक्र $x = \sqrt{y}$ तथा y-अक्ष के बीच घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and y-axis.

22. यदि $xy = c^2$ है, तो $(ax + by)$ का न्यूनतम मान ज्ञात कीजिए ।

अथवा

परवलय $y = x^2 + 7x + 2$ पर एक ऐसा बिन्दु ज्ञात कीजिए जो सरल रेखा $y = 3x - 3$ से न्यूनतम दूरी पर हो ।

Find the minimum value of $(ax + by)$, where $xy = c^2$.

OR

Find the coordinates of a point of the parabola $y = x^2 + 7x + 2$ which is closest to the straight line $y = 3x - 3$.

23. निम्न अवरोधों के अन्तर्गत $z = 8x + 9y$ का अधिकतमीकरण कीजिए :

$$2x + 3y \leq 6$$
$$3x - 2y \leq 6$$
$$y \leq 1$$
$$x, y \geq 0$$

Maximise $z = 8x + 9y$ subject to the constraints given below :

$$2x + 3y \leq 6$$
$$3x - 2y \leq 6$$
$$y \leq 1$$
$$x, y \geq 0$$

24. समतल $x - y + z = 5$ से बिन्दु $(1, -2, 3)$ की वह दूरी ज्ञात कीजिए, जो उस रेखा के समान्तर है, जिसके दिक्-कोसाइन $2, 3, -6$ के समानुपाती हैं ।

Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line whose direction cosines are proportional to $2, 3, -6$.

25. निम्नलिखित अवकल समीकरण का हल ज्ञात कीजिए :

$$\left[y - x\cos\left(\frac{y}{x}\right)\right]dy + \left[y\cos\left(\frac{y}{x}\right) - 2x\sin\left(\frac{y}{x}\right)\right]dx = 0$$

अथवा

निम्नलिखित अवकल समीकरण को हल कीजिए :

$$\left(\sqrt{1+x^2+y^2+x^2y^2}\right)dx + xy\,dy = 0$$

Solve the following differential equation :

$$\left[y - x\cos\left(\frac{y}{x}\right)\right]dy + \left[y\cos\left(\frac{y}{x}\right) - 2x\sin\left(\frac{y}{x}\right)\right]dx = 0$$

OR

Solve the following differential equation :

$$\left(\sqrt{1+x^2+y^2+x^2y^2}\right)dx + xy\,dy = 0$$

26. पासों के एक जोड़े को चार बार उछालने पर द्विकों की संख्या का प्रायिकता बंटन ज्ञात कीजिए । इस बंटन का माध्य तथा प्रसरण भी ज्ञात कीजिए ।

Find the probability distribution of the number of doublets in four throws of a pair of dice. Also find the mean and variance of this distribution.

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Senior School Certificate Examination

March — 2015

Marking Scheme — Mathematics 65/1/F, 65/2/F, 65/3/F

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 65/1/F

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\vec{a} + \vec{b} = 6\hat{i} + \hat{k}$ ½ m

$\therefore$ Reqd. unit vector $= \frac{6}{\sqrt{37}}\hat{i} + \frac{1}{\sqrt{37}}\hat{k}$ ½ m

2. Reqd. area $= \left|\vec{a} \times \vec{b}\right|$ ½ m

$\therefore \left|12\hat{i} - 4\hat{j} + 8\hat{k}\right| = \sqrt{144+16+64} = \sqrt{224}$ or $4\sqrt{14}$ sq. units ½ m

3. Getting x – intercept $= \frac{5}{2}$, y – intercept $= 5$, z – intercept $= -5$ ½ m

$\therefore$ Their sum $= \frac{5}{2}$ ½ m

4. co – factor of $a_{21} = 3$ 1 m

5. Degree = Order = 2 any one correct ½ m

$\therefore$ Degree + order = 4 ½ m

6. $2^y\, dy = dx \Rightarrow \frac{2^y}{\log 2} = x + c$ ½+½ m

SECTION - B

7. Getting $A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$ 1 m

$4A - 3I = \begin{bmatrix} 8-3 & -4 \\ -4 & 8-3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} = A^2$ 1 m

$A^2 = 4A - 3I$ ....(i)

Multiply both sides by A^{-1} ½ m

$A = 4I - 3A^{-1}$ or $A^{-1} = \frac{1}{3}(4I - A) = \frac{1}{3}\begin{pmatrix} 4-2 & 0+1 \\ 0+1 & 4-2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ 1½ m

OR

$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a^2+b & a-1 \\ b(a-1) & b+1 \end{bmatrix}$ 1½ m

$(A+B)^2 = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} (1+a)^2 & 0 \\ (2+b)(1+a) - 2(2+b) & 4 \end{pmatrix}$ .... (i) 1½ m

$A^2 + B^2 = \begin{pmatrix} a^2+b-1 & a-1 \\ b(a-1) & b \end{pmatrix}$ ....(ii)

Equating (i) and (ii), we get $b = 4, a = 1$ 1 m

8. Using $C_1 \to C_1 + C_2 + C_3$ and taking $a^2 + a + 1$ common from C_1

$\Delta = (a^2 + a + 1)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix}$, using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ 1½ m

$$\Delta = \left(a^2+a+1\right)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1-a & a(1-a) \\ 0 & a(a-1) & (1-a)(1+a) \end{vmatrix} = \left(a^2+a+1\right)(1-a)^2\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 0 & -a & 1+a \end{vmatrix}$$ 1½ m

$$\left.\begin{aligned} &= \left(a^2+a+1\right)(1-a)^2\left(1+a+a^2\right) \\ &= \left[(1-a)\left(1+a+a^2\right)\right]^2 = \left(1-a^3\right)^2 \end{aligned}\right\}$$ 1 m

9. Let $x+a=t \Rightarrow dx=dt$ and $x=t-a \Rightarrow x-a=t-2a$ 1 m

$$\therefore\ I = \int \frac{\sin(t-2a)\,dt}{\sin t} = \int \frac{(\sin t\cos 2a - \cos t\cdot \sin 2a)\,dt}{\sin t}$$ 1 m

$$= \cos 2a \int dt - \sin 2a \int \cot t\, dt = \cos 2a\ t - \sin 2a\,.\log|\sin t| + c$$ 1 m

$$= \cos 2a\,(x+a) - \sin 2a \log|\sin(x+a)| + c$$ 1

OR

consider $\dfrac{x^2}{\left(x^2+4\right)\left(x^2+9\right)}\cdot$ Let $x^2=t$ ½ m

$$\therefore\ \frac{t}{(t+4)(t+9)} = -\frac{4}{5}\ \frac{1}{t+4} + \frac{9}{5}\ \frac{1}{t+9}$$ 1 m

$$I = \int \frac{t\quad dt}{(t+4)(t+9)} = -\frac{4}{5}\int\frac{dt}{t+4} + \frac{9}{5}\int\frac{dt}{t+9}$$ ½ m

$$= \frac{-4}{5}\log|t+4| + \frac{9}{5}\log|t+9| + c$$ 1½ m

$$\therefore\ I = -\frac{4}{5}\log\left|x^2+4\right| + \frac{9}{5}\log\left|x^2+9\right| + c$$ ½ m

10. Writing given integral as 1 m

$$I = \int_{-\pi/2}^{0}\frac{\cos x}{1+e^x}\,dx + \int_{0}^{\pi/2}\frac{\cos x}{1+e^x}\,dx$$ Let $x=-t$, $dx=-dt$ 1½ m

when $x=-\dfrac{\pi}{2}$, $t=\pi/2$

$x=0$, $t=0$

$$\therefore\ I = \int_0^{\frac{\pi}{2}} \frac{e^t \cos t}{1+e^t}\, dt + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\, dx = \int_0^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^x}\, dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\, dx$$ 1½ m

$$I = \int_0^{\frac{\pi}{2}} \frac{\left(1+e^x\right)\cos x\, dx}{\left(1+e^x\right)} = \int_0^{\frac{\pi}{2}} \cos x\, dx = \left(\sin\ x\right)_0^{\frac{\pi}{2}} = 1$$ 1 m

11. Let B_1, B_2, B_3 be the events that the bolts produced by machines E_1, E_2, E_3 and A be the event that the selected bulb is defective ½ m

$$\left.\begin{array}{l} \therefore\ P\left(B_1\right) = \frac{1}{2},\ P\left(B_2\right) = P\left(B_3\right) = \frac{1}{4} \\ \\ P\left(A/B_1\right) = \frac{1}{25},\ P\left(A/B_2\right) = \frac{1}{25},\ P\left(A/B_3\right) = \frac{1}{20} \end{array}\right\}$$ 1½ m

$$P\left(A\right) = \sum_{c=1}^{3} P\left(B_c\right) P\left(A/B_c\right) = \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20} + \frac{1}{4} \times \frac{1}{25}$$ 1+½ m

$$= \frac{17}{400}$$ ½ m

OR

$$\therefore\ P\left(x=3\right) = \frac{1}{6} \times \frac{1}{5} \times 2 = \frac{1}{15},\ P\left(x=4\right) = \frac{2}{6} \times \frac{1}{5} \times 2 = \frac{2}{15}$$

Simlarly $P\left(x=5\right) = \frac{3}{15},\ P\left(x=6\right) = \frac{4}{15},\ P\left(x=7\right) = \frac{5}{15}$

Prob. distribution is

x :	3	4	5	6	7
P (x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

2 m

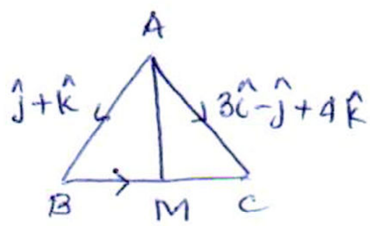
$x \cdot P(x):\quad \frac{3}{15} \quad \frac{8}{15} \quad \frac{15}{15} \quad \frac{24}{15} \quad \frac{35}{15}$

$x^2 P(x):\quad \frac{9}{15} \quad \frac{32}{15} \quad \frac{75}{15} \quad \frac{144}{15} \quad \frac{245}{15}$

$$\text{Mean} = \sum x_i \cdot P(x_i) = \frac{85}{15} = \frac{17}{3}$$ 1 m

$$\text{Variance} = \sum x_i^2\, P(x_i) - (\text{Mean})^2 = \frac{101}{3} - \frac{289}{9} = \frac{14}{9}$$ 1 m

12.



$$\vec{BC} = \left(3\hat{i} - \hat{j} + 4\hat{k}\right) - \left(\hat{j} + \hat{k}\right) = 3\hat{i} - 2\hat{j} + 3\hat{k}$$ 1½ m

$$\therefore\ \vec{BM} = \frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k}$$ 1 m

$$AM = \left|\hat{j} + \hat{k} + \frac{3\hat{i} - 2\hat{j} + 3\hat{k}}{2}\right| = \left|\frac{3\hat{i} + 5\hat{k}}{2}\right| = \frac{\sqrt{34}}{2}$$ 1½ m

13. Any plane through given point is $a(x-3) + b(y-6) + c(z-4) = 0$..........(i) 1 m

with $a + 5b + 4c = 0$ ....(A) ½ m

(i) passes through $(3, 2, 0)$ $\Rightarrow -4b - 4c = 0$ or $b + c = 0$ ....(B) ½ m

From (A) and (B) $a + b + (4b + 4c) = 0 \Rightarrow a = -b$ 1 m

$$\left.\begin{array}{l} \therefore\ a = -b = c \\ \therefore\ \text{Required eqn. of plane is } x - y + z - 1 = 0 \end{array}\right\}$$ 1 m

14. $\text{LHS} = \tan^{-1}\left(\dfrac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}\dfrac{2\cos\theta}{\sin^2\theta}$ 2 m

$$\therefore\ \tan^{-1}\frac{2\cos\theta}{\sin^2\theta} = \tan^{-1}\left(\frac{2}{\sin\theta}\right)$$ 1 m

$$\Rightarrow \cot\theta = 1 \text{ or } \theta = \pi/4$$ 1 m

OR

The given equation can be written

$$\left(\tan^{-1}2-\tan^{-1}1\right)+\left(\tan^{-1}3-\tan^{-1}2\right)+\left(\tan^{-1}4-\tan^{-1}3\right)+.....+\tan^{-1}(n+1)-\tan^{-1}n = \tan^{-1}\theta$$ 2 m

$$\Rightarrow \tan^{-1}(n+1)-\tan^{-1}1 = \tan^{-1}\theta$$ 1 m

$$\Rightarrow \tan^{-1}\frac{n+1-1}{1+(n+1)} = \tan^{-1}\theta \Rightarrow \theta = \frac{n}{n+2}$$ 1 m

15. $9y^2 = x^3 \Rightarrow 18y\dfrac{dy}{dx} = 3x^2 \Rightarrow \dfrac{dy}{dx} = \text{slope of tangent} = \dfrac{x^2}{6y}$

$\therefore$ Slope of normal $= -\dfrac{6y}{x^2}$ 1+½ m

As the intercepts by normal on both axes are equal

$\therefore$ Slope of normal $= \pm 1 \Rightarrow \dfrac{-6y}{x^2} = \pm 1 \Rightarrow y = \pm\dfrac{x^2}{6}$ 1 m

$$\therefore\ 9\left(\frac{x^4}{36}\right) = x^3 \Rightarrow x = 4 \text{ and } y^2 = \frac{64}{9} \Rightarrow y = \pm 8/3$$ 1 m

$\therefore$ The points are $\left(4, \dfrac{8}{3}\right)$, $\left(4, -8/3\right)$ ½ m

16. $$\frac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^{n-1}\left[1+\frac{x}{\sqrt{1+x^2}}\right] = \frac{n}{\sqrt{1+x^2}}\left[x+\sqrt{1+x^2}\right]^n = \frac{ny}{\sqrt{1+x^2}}$$ 1½ m

$$\therefore\ \sqrt{1+x^2}\,\frac{dy}{dx} = ny \text{...........(i)}$$ ½ m

$$\therefore\ \sqrt{1+x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{x}{\sqrt{1+x^2}} = n\frac{dy}{dx}$$ 1 m

$$\Rightarrow\ \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n\sqrt{1+x^2}\,\frac{dy}{dx} = n\cdot ny \ \text{(from (i))}$$

$$= n^2 y$$ 1 m

17. L H D at $x = 1$: $\lim\limits_{x\to 1^-}\left(\frac{x-1}{x-1}\right) = 1$

R H D at $x = 1$, $\lim\limits_{x\to 1^+}\frac{2-x-1}{x-1} = -1$ 2 m

$\therefore$ f is not differentiable at $x = 1$

L H D at $x = 2$, $\lim\limits_{x\to 2^-}\frac{2-x-0}{x-2} = -1$

R H D at $x = 2$, $\lim\limits_{x\to 2^+}\frac{-2+3x-x^2}{(x-2)} = \lim\limits_{x\to 2^+} - \frac{(x-1)(x-2)}{(x-2)} = -1$ 2 m

$\therefore$ f is diff. at $x = 2$

18. Communication Matrix $A = \begin{array}{c}\begin{pmatrix}140\\200\\150\end{pmatrix}\end{array}\begin{array}{l}\text{Telephone}\\\text{House calls}\\\text{Letters}\end{array}$

$$\begin{array}{ccc} \text{Tele} & \text{House calls} & \text{Letters} \end{array}$$

$$\text{Cost Matrix } B = \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} \begin{array}{l} \text{City x} \\ \text{City y} \end{array}$$

$$\therefore \text{ Total cost Matrix } = \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} \begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} = \begin{pmatrix} 990000 \\ 2120000 \end{pmatrix}$$ 3 m

any relevant value 1 m

19. $I = \int e^{2x} \sin(3x+1)\,dx = \left[\frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \int e^{2x} \cos(3x+1)\,dx\right]$ 1½ m

$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \cdot \frac{e^{2x}}{2} \cdot \cos(3x+1) - \frac{9}{4} \int e^{2x} \sin(3x+1)\,dx \Big]$ 1 m

$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1) - \frac{9}{4} I$ 1 m

$$\left.\begin{aligned} \frac{13}{4} I &= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1) \\ I &= \frac{4}{13}\left[\frac{e^{2x}}{2}\left(-\frac{3}{2}\cos(3x+1) + \sin(3x+1)\right)\right] + c \end{aligned}\right\}$$ ½ m

SECTION - C

20. Let $x_1, x_2, \in R$ such that $f(x_1) = f(x_2) \Rightarrow 4x_1^{\ 2} + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$ 1½+½ m

$\Rightarrow 4(x_1 - x_2)[x_1 + x_2 + 3] = 0 \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one – one

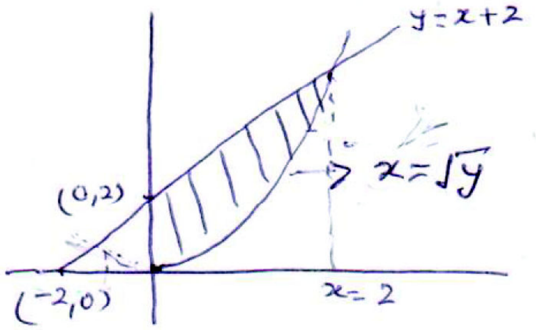
f is clearly onto and hence intertible 1 m

Let y be an arbitrary element of S

$f(x) = y = 4x^2 + 12x + 15 = (2x+3)^2 + 6$ 1 m

$\therefore f^{-1} : R \to S$ is given by $f^{-1}(y) = \left(\frac{\sqrt{y-6} - 3}{2}\right)$ 2 m

21.



Correct Figure 1m

Points of intersection

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$ (-1 is rejected) $\Big\}$ 1½ m

$\therefore$ Reqd. area $= \int_0^2 \left\{(x+2) - x^2\right\} dx$ 1½ m

$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_0^2$

$= \left(2 + 4 - \frac{8}{3}\right) = \frac{10}{3}$ sq.units 2 m

22. Let $z = ax + by$, also $xy = c^2 \Rightarrow y = \frac{c^2}{x}$

$\therefore \; z = ax + \frac{bc^2}{x}$ $\Big\}$ ½ + ½ m

$\therefore \frac{dz}{dx} = a + bc^2\left(\frac{-1}{x^2}\right), \; \frac{dz}{dx} = 0 \Rightarrow bc^2 = ax^2$

or $x = \sqrt{\frac{b}{a}}\, c$ 1½ m

showing $\frac{d^2z}{ax^2}$ at $x = \sqrt{\frac{b}{a}}\, c > 0 \Rightarrow$ minima 1½ m

$y = \frac{c^2}{x} = \frac{c^2}{c}\sqrt{\frac{a}{b}} = c\sqrt{\frac{a}{b}}$ 1 m

$\therefore$ minimum $z = a\sqrt{\frac{b}{a}}\, c + bc\sqrt{\frac{a}{b}}\, c = 2\sqrt{ab}\, c$ 1 m

OR

$y = x^2 + 7x + 2\ ,\ 3x - y - 3 = 0$ ....(i)

$\therefore\ 3x - (x^2 + 7x + 2) - 3 = 0$ 1 m

Distance of (x, y) from (i)

$D = \left| \dfrac{3x - (x^2 + 7x + 2) - 3}{\sqrt{10}} \right|$ or $D = \left| \dfrac{(-x^2 - 4x - 5)}{\sqrt{10}} \right| = \left| \dfrac{(x+2)^2 + 1}{\sqrt{10}} \right|$ 2 m

$\dfrac{dD}{dx} = \dfrac{2}{\sqrt{10}}(x+2),\ \dfrac{dD}{dx} = 0$ at $x = -2$ 1 m

$\dfrac{d^2D}{dx^2} > 0 \Rightarrow$ minima 1 m

$\therefore$ D is minimum at $x = -2$

at $x = -2,\ y = -8$

$\therefore$ The required pt. on the parabola is $(-2, -8)$ 1 m

23.

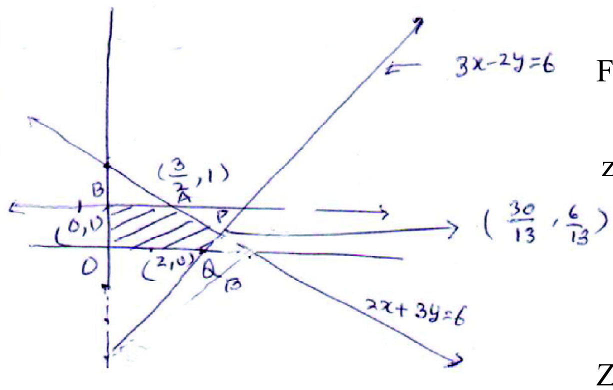


Figure 3 m

Feasible region is B A PQO 1 m

$z_B = 9,\ z_P = \dfrac{240}{13} + \dfrac{54}{13} = \dfrac{294}{13}$

$= 22\dfrac{8}{13}$ 1 m

$Z_Q = 16$

$\therefore$ Z is maximum at $\left(\dfrac{30}{13}, \dfrac{6}{13}\right)$

and maximum value $= 22\dfrac{8}{13}$ 1 m

24. Any line through $(1,-2,3)$ with d. r's as 2, 3 – 6 is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$$ 1 ½ m

$$\therefore\ x = 2\lambda+1,\ y = 3\lambda-2,\ z = -6\lambda+3$$ 1½ m

It lies on the plane $x-y+z=5$

$$\therefore\ 2\lambda+1-3\lambda+2-6\lambda+3 = 5$$

$$\Rightarrow\ \lambda = \frac{1}{7}$$ 1 m

Reqd. point is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$ 1 m

$$\therefore\ \text{Reqd distance} = \sqrt{\left(\frac{9}{7}-1\right)^2+\left(-\frac{11}{7}+2\right)^2+\left(\frac{15}{7}-3\right)^2} = \frac{1}{7}\sqrt{4+9+36} = \frac{7}{7} = 1$$ 1 m

25. $$\frac{dy}{dx} = \frac{2x\sin\left(\frac{y}{x}\right) - y\cos\left(\frac{y}{x}\right)}{y-x\cos\left(\frac{y}{x}\right)} = \frac{2\sin\left(\frac{y}{x}\right) - \frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{y}{x}-\cos\left(\frac{y}{x}\right)}$$ 1 m

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 m

$$\therefore\ v + x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v-\cos v} \Rightarrow x\frac{dv}{dx} = \frac{2\sin v - v^2}{v-\cos v}$$ 1 m

$$\Rightarrow \frac{v-\cos v}{-2\sin v+v^2} = \frac{-dx}{x} \text{ or } \frac{1}{2}\left[\frac{2v-2\cos v}{-2\sin v+v^2}\right]dv = -\frac{dx}{x}$$ 1 m

$$\left.\begin{aligned}&\Rightarrow \ \frac{1}{2}\ \log\left|v^2 - 2\sin v\right| = -\log x + \log c\\ &\text{or } \log\left|\sqrt{v^2 - 2\sin v}\right| = \log c - \log x\end{aligned}\right\}$$ 1 m

$$\sqrt{v^2 - 2\sin v} = \frac{c}{x}$$

$$\text{or } x\sqrt{\frac{y^2}{x^2} - 2\sin\frac{y}{x}} = c$$ ½ m

$$y^2 - 2x^2 \sin\left(\frac{y}{x}\right) = c'$$ ½ m

OR

$$\left(\sqrt{\left(1+x^2\right)\left(1+y^2\right)}\right)\, dx + xy\, dy = 0$$

$$\Rightarrow \ \frac{y}{\sqrt{1+y^2}}\ dy + \frac{\sqrt{1+x^2}}{x}\ dx = 0$$ 1 m

$$\frac{1}{2}\int \frac{2y}{\sqrt{1+y^2}}\, dy + \int \frac{\sqrt{1+x^2}}{x}\, dx = 0$$

$$\sqrt{1+y^2} + \int \frac{\left(1+x^2\right)}{x\sqrt{1+x^2}}\, dx = c$$ 1½ m

$$I_1 = \int \frac{1}{x\sqrt{1+x^2}}\ dx + \int \frac{x}{\sqrt{1+x^2}}\ dx = I_2 + \sqrt{1+x^2}$$ 1 m

For I_2, Let $x = \frac{1}{t}$, $dx = \frac{-1}{t^2}\, dt$ 1 m

$$I_2 = \int \frac{-1}{t^2 \cdot \frac{1}{t}\sqrt{1+\frac{1}{t^2}}}\ dt = -\int \frac{dt}{\sqrt{t^2+1}} = -\log\left[t + \sqrt{1+t^2}\right]$$ 1 m

$$= -\log\left[\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right] = -\log\left[\frac{1+\sqrt{1+x^2}}{x}\right]$$

$\therefore$ The solution is $\sqrt{1+x^2}+\sqrt{1+y^2} - \log\left(\frac{1+\sqrt{1+x^2}}{2}\right) = c$ ½ m

26. $P(\text{Doublet}) = \frac{1}{6}$, $P(\text{not a doublet}) = 5/6$

The random variate x can take values 0, 1, 2, 3, 4 1 m

x	0	1	2	3	4	
P(x)	$\left(\frac{5}{6}\right)^4$	$4\ \frac{1}{6}\left(\frac{5}{6}\right)^3$	$6\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$	$4\left(\frac{1}{6}\right)^3\frac{5}{6}$	$\left(\frac{1}{6}\right)^4$	
	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$	2½ m

Mean $= \sum x\ P(x) = \frac{500+300+60+4}{1296} = \frac{864}{1296} = \frac{2}{3}$ 1 m

$\sum x^2\ P(x) = \frac{500+600+180+16}{1296} = \frac{1296}{1296} = 1$ 1 m

$\therefore$ Variance $= 1-\left(\frac{2}{3}\right)^2 = \frac{5}{9}$ ½ m

QUESTION PAPER CODE 65/2/F

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

			Marks
1.	$\text{co}-\text{factor of } a_{21} = 3$		1 m
2.	$\text{Degree} = \text{Order} = 2$	any one correct	½ m
	$\therefore\ \text{Degree} + \text{order} = 4$		½ m
3.	$2^y\ dy = dx \Rightarrow \dfrac{2^y}{\log 2} = x + c$		½+½ m
4.	$\vec{a} + \vec{b} = 6\hat{i} + \hat{k}$		½ m
	$\therefore$ Reqd. unit vector $= \dfrac{6}{\sqrt{37}}\hat{i} + \dfrac{1}{\sqrt{37}}\hat{k}$		½ m
5.	Reqd. area $= \left\lvert \vec{a} \times \vec{b} \right\rvert$		½ m
	$\therefore\ \left\lvert 12\hat{i} - 4\hat{j} + 8\hat{k} \right\rvert = \sqrt{144+16+64} = \sqrt{224}$ or $4\sqrt{14}$ sq. units		½ m
6.	Getting $x-\text{intercept} = \dfrac{5}{2},\ y-\text{intercept} = 5,\ z-\text{intercept} = -5$		½ m
	$\therefore$ Their sum $= \dfrac{5}{2}$		½ m

SECTION - B

7. Writing given integral as 1 m

$$I = \int_{-\pi/2}^{0} \frac{\cos x}{1+e^x}\,dx + \int_{0}^{\pi/2} \frac{\cos x}{1+e^x}\,dx$$

Let $x = -t$, $dx = -dt$ 1½ m

when $x = -\frac{\pi}{2}$, $t = \pi/2$

$x = 0$, $t = 0$

$$\therefore\ I = \int_0^{\frac{\pi}{2}} \frac{e^t \cos t}{1+e^t}\,dt + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\,dx = \int_0^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^x}\,dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\,dx$$ 1½ m

$$I = \int_0^{\frac{\pi}{2}} \frac{\left(1+e^x\right)\cos x\,dx}{\left(1+e^x\right)} = \int_0^{\frac{\pi}{2}} \cos x\,dx = \left(\sin\ x\right)_0^{\frac{\pi}{2}} = 1$$ 1 m

8. Let B_1, B_2, B_3 be the events that the bolts produced by machines E_1, E_2, E_3 and A be the event that the selected bulb is defective ½ m

$$\therefore\ P\left(B_1\right) = \frac{1}{2},\ P\left(B_2\right) = P\left(B_3\right) = \frac{1}{4}$$

1½ m

$$P\left(A/B_1\right) = \frac{1}{25},\ P\left(A/B_2\right) = \frac{1}{25},\ P\left(A/B_3\right) = \frac{1}{20}$$

$$P\left(A\right) = \sum_{c=1}^{3} P\left(B_c\right) P\left(A/B_c\right) = \frac{1}{2}\times\frac{1}{25}+\frac{1}{4}\times\frac{1}{20}+\frac{1}{4}\times\frac{1}{25}$$ 1+½ m

$$= \frac{17}{400}$$ ½ m

OR

$$\therefore\ P(x=3)=\frac{1}{6}\times\frac{1}{5}\times 2=\frac{1}{15},\ P(x=4)=\frac{2}{6}\times\frac{1}{5}\times 2=\frac{2}{15}$$

Simlarly $P(x=5)=\frac{3}{15},\ P(x=6)=\frac{4}{15},\ P(x=7)=\frac{5}{15}$

Prob. distridution is

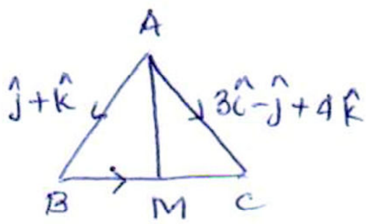
x :	3	4	5	6	7
P (x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$
x . P (x) :	$\frac{3}{15}$	$\frac{8}{15}$	$\frac{15}{15}$	$\frac{24}{15}$	$\frac{35}{15}$
x^2 P (x) :	$\frac{9}{15}$	$\frac{32}{15}$	$\frac{75}{15}$	$\frac{144}{15}$	$\frac{245}{15}$

2 m

Mean $=\sum x_1\cdot P(x_1)=\frac{85}{15}=\frac{17}{3}$ 1 m

Variance $=\sum x_i^2\ P(x_i)-(\text{Mean})^2=\frac{101}{3}-\frac{289}{9}=\frac{14}{9}$ 1 m

9.



$\vec{BC}=\left(3\hat{i}-\hat{j}+4\hat{k}\right)-\left(\hat{j}+\hat{k}\right)=3\hat{i}-2\hat{j}+3\hat{k}$ 1½ m

$\therefore\ \vec{BM}=\frac{3}{2}\hat{i}-\hat{j}+\frac{3}{2}\hat{k}$ 1 m

$AM=\left|\hat{j}+\hat{k}+\frac{3\hat{i}-2\hat{j}+3\hat{k}}{2}\right|=\left|\frac{3\hat{i}+5\hat{k}}{2}\right|=\frac{\sqrt{34}}{2}$ 1½ m

10. Any plane through given point is $a(x-3)+b(y-6)+c(z-4)=0$..........(i) 1 m

with $a+5b+4c=0$ ....(A) ½ m

(i) passes through (3, 2, 0) $\Rightarrow -4b - 4c = 0$ or $b + c = 0$ ....(B) ½ m

From (A) and (B) $a + b + (4b + 4c) = 0 \Rightarrow a = -b$ 1 m

$\therefore\ a = -b = c$ 1 m

$\therefore$ Required eqn. of plane is $x - y + z - 1 = 0$

11. LHS $= \tan^{-1}\left(\dfrac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}\dfrac{2\cos\theta}{\sin^2\theta}$ 2 m

$\therefore\ \tan^{-1}\dfrac{2\cos\theta}{\sin^2\theta} = \tan^{-1}\left(\dfrac{2}{\sin\theta}\right)$ 1 m

$\Rightarrow \cot\theta = 1$ or $\theta = \pi/4$ 1 m

OR

The given equation can be written

$(\tan^{-1}2 - \tan^{-1}1) + (\tan^{-1}3 - \tan^{-1}2) + (\tan^{-1}4 - \tan^{-1}3) + \ldots.. + \tan^{-1}(n+1) - \tan^{-1}n = \tan^{-1}\theta$ 2 m

$\Rightarrow \tan^{-1}(n+1) - \tan^{-1}1 = \tan^{-1}\theta$ 1 m

$\Rightarrow \tan^{-1}\dfrac{n+1-1}{1+(n+1)} = \tan^{-1}\theta \Rightarrow \theta = \dfrac{n}{n+2}$ 1 m

12. Getting $A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$ 1 m

$4A - 3I = \begin{bmatrix} 8-3 & -4 \\ -4 & 8-3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} = A^2$ 1 m

$A^2 = 4A - 3I$ .... (i)

Multiply both sides by A^{-1} ½ m

$$A = 4I - 3A^{-1} \text{ or } A^{-1} = \frac{1}{3}(4I - A) = \frac{1}{3}\begin{pmatrix} 4-2 & 0+1 \\ 0+1 & 4-2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$ 1½ m

OR

$$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},\ B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a^2+b & a-1 \\ b(a-1) & b+1 \end{bmatrix}$$ 1½ m

$$(A+B)^2 = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} (1+a)^2 & 0 \\ (2+b)(1+a) - 2(2+b) & 4 \end{pmatrix} \text{ .... (i)}$$ 1½ m

$$A^2 + B^2 = \begin{pmatrix} a^2+b-1 & a-1 \\ b(a-1) & b \end{pmatrix} \text{ ....(ii)}$$

Equating (i) and (ii), we get $b = 4, a = 1$ 1 m

13. Using $C_1 \to C_1 + C_2 + C_3$ and taking $a^2 + a + 1$ common from C_1

$$\Delta = (a^2+a+1)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix},\ \text{using } R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$$ 1½ m

$$\Delta = (a^2+a+1)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1-a & a(1-a) \\ 0 & a(a-1) & (1-a)(1+a) \end{vmatrix} = (a^2+a+1)(1-a)^2\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 0 & -a & 1+a \end{vmatrix}$$ 1½ m

$$\left.\begin{aligned} &= (a^2+a+1)(1-a)^2(1+a+a^2) \\ &= [(1-a)(1+a+a^2)]^2 = (1-a^3)^2 \end{aligned}\right\}$$ 1 m

14. Let $x+a=t \Rightarrow dx=dt$ and $x=t-a \Rightarrow x-a=t-2a$ 1 m

$$\therefore\ I = \int \frac{\sin(t-2a)\,dt}{\sin t} = \int \frac{(\sin t \cos 2a - \cos t \cdot \sin 2a)\,dt}{\sin t}$$ 1 m

$$= \cos 2a \int dt - \sin 2a \int \cot t\, dt = \cos 2a\, t - \sin 2a \,.\, \log|\sin t| + c$$ 1 m

$$= \cos 2a\,(x+a) - \sin 2a \log|\sin(x+a)| + c$$ 1

OR

consider $\dfrac{x^2}{(x^2+4)(x^2+9)}$ · Let $x^2 = t$ ½ m

$$\therefore\ \frac{t}{(t+4)(t+9)} = -\frac{4}{5}\ \frac{1}{t+4} + \frac{9}{5}\ \frac{1}{t+9}$$ 1 m

$$I = \int \frac{t\quad dt}{(t+4)(t+9)} = -\frac{4}{5}\int \frac{dt}{t+4} + \frac{9}{5}\int \frac{dt}{t+9}$$ ½ m

$$= \frac{-4}{5}\ \log\ |t+4| + \frac{9}{5}\log|t+9| + c$$ 1½ m

$$\therefore\ I = -\frac{4}{5}\ \log|x^2+4| + \frac{9}{5}\log|x^2+9| + c$$ ½ m

15. L H D at $x=1$: $\lim\limits_{x\to 1^-} \left(\dfrac{x-1}{x-1}\right) = 1$

2 m

R H D at $x=1$, $\lim\limits_{x\to 1^+} \dfrac{2-x-1}{x-1} = -1$

$\therefore$ f is not differentiable at $x=1$

L H D at $x=2$, $\lim\limits_{x\to 2^-} \dfrac{2-x-0}{x-2} = -1$

2 m

R H D at $x=2$, $\lim\limits_{x\to 2^+} \dfrac{-2+3x-x^2}{(x-2)} = \lim\limits_{x\to 2^+} -\dfrac{(x-1)(x-2)}{(x-2)} = -1$

$\therefore$ f is diff. at $x=2$

16. Communication Matrix $A = \begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} \begin{array}{c} \text{Telephone} \\ \text{House calls} \\ \text{Letters} \end{array}$

$$\text{Cost Matrix } B = \begin{array}{c} \begin{array}{ccc} \text{Tele} & \text{House calls} & \text{Letters} \end{array} \\ \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} \end{array} \begin{array}{l} \\ \text{City x} \\ \text{City y} \end{array}$$

$\therefore$ Total cost Matrix $= \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} \begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} = \begin{pmatrix} 990000 \\ 2120000 \end{pmatrix}$ 3 m

any relevant value 1 m

17. $I = \int e^{2x} \sin(3x+1)\,dx = \left[\frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \int e^{2x} \cos(3x+1)\,dx\right]$ 1½ m

$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \cdot \frac{e^{2x}}{2} \cdot \cos(3x+1) - \frac{9}{4} \int e^{2x} \sin(3x+1)\,dx\,\Big]$ 1 m

$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1) - \frac{9}{4} I$ 1 m

$\frac{13}{4} I = \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1)$

½ m

$I = \frac{4}{13}\left[\frac{e^{2x}}{2}\left(-\frac{3}{2}\cos(3x+1) + \sin(3x+1)\right)\right] + c$

18. $9y^2 = x^3 \Rightarrow 18y \frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \text{slope of tangent} = \frac{x^2}{6y}$

1+½ m

$\therefore$ Slope of normal $= -\frac{6y}{x^2}$

As the intercepts by normal on both axes are equal

1 m

$\therefore$ Slope of normal $= \pm 1 \Rightarrow \frac{-6y}{x^2} = \pm 1 \Rightarrow y = \pm \frac{x^2}{6}$

$\therefore \quad 9\left(\frac{x^4}{36}\right) = x^3 \Rightarrow x = 4$ and $y^2 = \frac{64}{9} \Rightarrow y = \pm 8/3$ 1 m

$\therefore$ The points are $\left(4, \frac{8}{3}\right), \left(4, -8/3\right)$ ½ m

19. $\frac{dy}{dx} = n\left(x + \sqrt{1+x^2}\right)^{n-1}\left[1 + \frac{x}{\sqrt{1+x^2}}\right] = \frac{n}{\sqrt{1+x^2}}\left[x + \sqrt{1+x^2}\right]^n = \frac{ny}{\sqrt{1+x^2}}$ 1½ m

$\therefore \quad \sqrt{1+x^2}\,\frac{dy}{dx} = ny$............(i) ½ m

$\therefore \quad \sqrt{1+x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{x}{\sqrt{1+x^2}} = n\frac{dy}{dx}$ 1 m

$\Rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n\sqrt{1+x^2}\,\frac{dy}{dx} = n \cdot ny$ (from (i))

$= n^2 y$ 1 m

SECTION - C

20. Let $z = ax + by$, also $xy = c^2 \Rightarrow y = \frac{c^2}{x}$ ½ + ½ m

$\therefore \quad z = ax + \frac{bc^2}{x}$

$\therefore \quad \frac{dz}{dx} = a + bc^2\left(\frac{-1}{x^2}\right), \frac{dz}{dx} = 0 \Rightarrow bc^2 = ax^2$

or $x = \sqrt{\frac{b}{a}}\,c$ 1½ m

showing $\frac{d^2z}{ax^2}$ at $x = \sqrt{\frac{b}{a}}\, c > 0 \Rightarrow$ minima 1½ m

$y = \frac{c^2}{x} = \frac{c^2}{c}\sqrt{\frac{a}{b}} = c\sqrt{\frac{a}{b}}$ 1 m

$\therefore$ minimum $z = a\sqrt{\frac{b}{a}}\, c + bc\sqrt{\frac{a}{b}}\, c = 2\sqrt{ab}\, c$ 1 m

OR

$y = x^2 + 7x + 2\ ,\ 3x - y - 3 = 0$ ....(i)

$\therefore\ 3x - \left(x^2 + 7x + 2\right) - 3 = 0$ 1 m

Distance of (x, y) from (i)

$D = \left|\frac{3x - \left(x^2 + 7x + 2\right) - 3}{\sqrt{10}}\right|$ or $D = \left|\frac{\left(-x^2 - 4x - 5\right)}{\sqrt{10}}\right| = \left|\frac{(x+2)^2 + 1}{\sqrt{10}}\right|$ 2 m

$\frac{dD}{dx} = \frac{2}{\sqrt{10}}\left(x + 2\right),\ \frac{dD}{dx} = 0$ at $x = -2$ 1 m

$\frac{d^2D}{dx^2} > 0 \Rightarrow$ minima 1 m

$\therefore$ D is minimum at $x = -2$

at $x = -2,\ y = -8$ 1 m

$\therefore$ The required pt. on the parabola is $\left(-2, -8\right)$

21.

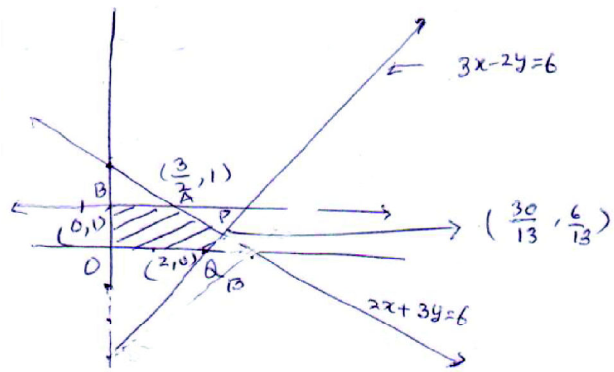


Figure — 3 m

Feasible region is B A PQO — 1 m

$z_B = 9,\ z_P = \frac{240}{13} + \frac{54}{13} = \frac{294}{13}$ — 1 m

$= 22\frac{8}{13}$

$Z_Q = 16$

$\therefore$ Z is maximum at $\left(\frac{30}{13}, \frac{6}{13}\right)$ — 1 m

and maximum value $= 22\frac{8}{13}$

22. Any line through $(1, -2, 3)$ with d. r's as 2, 3 − 6 is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda \qquad 1\tfrac{1}{2}\text{ m}$$

$$\therefore\ x = 2\lambda + 1,\ y = 3\lambda - 2,\ z = -6\lambda + 3 \qquad 1\tfrac{1}{2}\text{ m}$$

It lies on the plane $x - y + z = 5$

$$\therefore\ 2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$$

$$\Rightarrow\ \lambda = \frac{1}{7} \qquad 1\text{ m}$$

Reqd. point is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$ — 1 m

$$\therefore\ \text{Reqd distance} = \sqrt{\left(\frac{9}{7} - 1\right)^2 + \left(-\frac{11}{7} + 2\right)^2 + \left(\frac{15}{7} - 3\right)^2} = \frac{1}{7}\sqrt{4 + 9 + 36} = \frac{7}{7} = 1 \qquad 1\text{ m}$$

23. Let $x_1, x_2, \in R$ such that $f(x_1) = f(x_2) \Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$ 1½+½ m

$\Rightarrow 4(x_1 - x_2)[x_1 + x_2 + 3] = 0 \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one – one

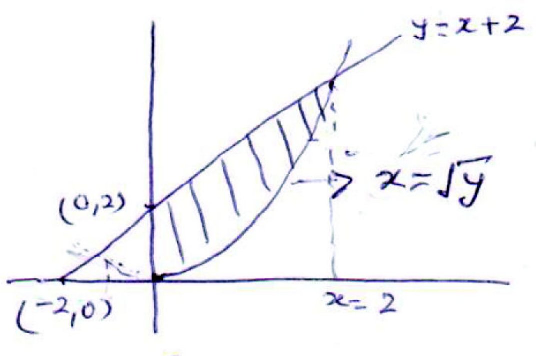
f is clearly onto and hence intertible 1 m

Let y be an arbitrary element of S

$f(x) = y = 4x^2 + 12x + 15 = (2x+3)^2 + 6$ 1 m

$\therefore f^{-1} : R \to S$ is given by $f^{-1}(y) = \left(\dfrac{\sqrt{y-6} - 3}{2}\right)$ 2 m

24.



Correct Figure 1m

Points of intersection

$x^2 - x - 2 = 0$ 1½ m

$(x-2)(x+1) = 0$

$x = 2, -1$ (– 1 is rejected)

$\therefore$ Reqd. area $= \int_0^2 \{(x+2) - x^2\} dx$ 1½ m

$= \left[\dfrac{x^2}{2} + 2x - \dfrac{x^3}{3}\right]_0^2$

$= \left(2 + 4 - \dfrac{8}{3}\right) = \dfrac{10}{3}$ sq.units 2 m

25. $P(\text{Doublet}) = \dfrac{1}{6}$, $P(\text{not a doublet}) = \dfrac{5}{6}$ 1 m

The random variate x can take values 0, 1, 2, 3, 4

x	0	1	2	3	4	
P(x)	$\left(\frac{5}{6}\right)^4$	$4\,\frac{1}{6}\left(\frac{5}{6}\right)^3$	$6\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$	$4\left(\frac{1}{6}\right)^3\frac{5}{6}$	$\left(\frac{1}{6}\right)^4$	
	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$	2½ m

$$\text{Mean} = \sum x\, P(x) = \frac{500+300+60+4}{1296} = \frac{864}{1296} = \frac{2}{3}$$ 1 m

$$\sum x^2\, P(x) = \frac{500+600+180+16}{1296} = \frac{1296}{1296} = 1$$ 1 m

$$\therefore\ \text{Variance} = 1-\left(\frac{2}{3}\right)^2 = \frac{5}{9}$$ ½ m

26. $$\frac{dy}{dx} = \frac{2x\sin\left(\frac{y}{x}\right) - y\cos\left(\frac{y}{x}\right)}{y - x\cos\left(\frac{y}{x}\right)} = \frac{2\sin\left(\frac{y}{x}\right) - \frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{y}{x} - \cos\left(\frac{y}{x}\right)}$$ 1 m

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$ 1 m

$$\therefore\ v + x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v - \cos v} \Rightarrow x\frac{dv}{dx} = \frac{2\sin v - v^2}{v - \cos v}$$ 1 m

$$\Rightarrow \frac{v-\cos v}{-2\sin v + v^2} = \frac{-dx}{x} \text{ or } \frac{1}{2}\left[\frac{2v - 2\cos v}{-2\sin v + v^2}\right]dv = -\frac{dx}{x}$$ 1 m

$$\Rightarrow \frac{1}{2}\log\left|v^2 - 2\sin v\right| = -\log x + \log c$$

$$\text{or } \log\left|\sqrt{v^2 - 2\sin v}\right| = \log c - \log x$$ 1 m

$$\sqrt{v^2 - 2\sin v} = \frac{c}{x}$$

$$\text{or } x\sqrt{\frac{y^2}{x^2} - 2\sin\frac{y}{x}} = c$$ ½ m

$$y^2 - 2x^2\sin\left(\frac{y}{x}\right) = c'$$ ½ m

OR

$$\left(\sqrt{(1+x^2)(1+y^2)}\right)\, dx + xy\, dy = 0$$

$$\Rightarrow \quad \frac{y}{\sqrt{1+y^2}}\, dy + \frac{\sqrt{1+x^2}}{x}\, dx = 0$$ 1 m

$$\frac{1}{2}\int \frac{2y}{\sqrt{1+y^2}}\, dy + \int \frac{\sqrt{1+x^2}}{x}\, dx = 0$$

$$\sqrt{1+y^2} + \int \frac{(1+x^2)}{x\sqrt{1+x^2}}\, dx = c$$ 1½ m

$$I_1 = \int \frac{1}{x\sqrt{1+x^2}}\, dx + \int \frac{x}{\sqrt{1+x^2}}\, dx = I_2 + \sqrt{1+x^2}$$ 1 m

For I_2, Let $x = \frac{1}{t}$, $dx = \frac{-1}{t^2}\, dt$ 1 m

$$I_2 = \int \frac{-1}{t^2 \cdot \frac{1}{t}\sqrt{1+\frac{1}{t^2}}}\, dt = -\int \frac{dt}{\sqrt{t^2+1}} = -\log\left[t+\sqrt{1+t^2}\right]$$ 1 m

$$= -\log\left[\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right] = -\log\left[\frac{1+\sqrt{1+x^2}}{x}\right]$$

½ m

$\therefore$ The solution is $\sqrt{1+x^2} + \sqrt{1+y^2} - \log\left(\frac{1+\sqrt{1+x^2}}{2}\right) = c$

QUESTION PAPER CODE 65/3/F

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. Reqd. area $= \left| \vec{a} \times \vec{b} \right|$ ½ m

$\therefore \quad \left| 12\,\hat{i} - 4\,\hat{j} + 8\,\hat{k} \right| = \sqrt{144+16+64} = \sqrt{224}$ or $4\sqrt{14}$ sq. units ½ m

2. Getting $x - \text{intercept} = \frac{5}{2}$, $y - \text{intercept} = 5$, $z - \text{intercept} = -5$ ½ m

$\therefore$ Their sum $= \frac{5}{2}$ ½ m

3. $\vec{a} + \vec{b} = 6\hat{i} + \hat{k}$ ½ m

$\therefore$ Reqd. unit vector $= \frac{6}{\sqrt{37}}\,\hat{i} + \frac{1}{\sqrt{37}}\,\hat{k}$ ½ m

4. Degree = Order = 2 any one correct ½ m

$\therefore$ Degree + order = 4 ½ m

5. $2^y\, dy = dx \Rightarrow \frac{2^y}{\log 2} = x + c$ ½+½ m

6. co – factor of $a_{21} = 3$ 1 m

SECTION - B

7. $9y^2 = x^3 \Rightarrow 18y\dfrac{dy}{dx} = 3x^2 \Rightarrow \dfrac{dy}{dx} = \text{slope of tangent} = \dfrac{x^2}{6y}$

$\therefore$ Slope of normal $= -\dfrac{6y}{x^2}$ 1+½ m

As the intercepts by normal on both axes are equal

$\therefore$ Slope of normal $= \pm 1 \Rightarrow \dfrac{-6y}{x^2} = \pm 1 \Rightarrow y = \pm\dfrac{x^2}{6}$ 1 m

$\therefore 9\left(\dfrac{x^4}{36}\right) = x^3 \Rightarrow x = 4$ and $y^2 = \dfrac{64}{9} \Rightarrow y = \pm 8/3$ 1 m

$\therefore$ The points are $\left(4, \dfrac{8}{3}\right), \left(4, -8/3\right)$ ½ m

8. $\dfrac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^{n-1}\left[1+\dfrac{x}{\sqrt{1+x^2}}\right] = \dfrac{n}{\sqrt{1+x^2}}\left[x+\sqrt{1+x^2}\right]^n = \dfrac{ny}{\sqrt{1+x^2}}$ 1½ m

$\therefore \sqrt{1+x^2}\dfrac{dy}{dx} = ny$..........(i) ½ m

$\therefore \sqrt{1+x^2}\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx}\cdot\dfrac{x}{\sqrt{1+x^2}} = n\dfrac{dy}{dx}$ 1 m

$\Rightarrow \left(1+x^2\right)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = n\sqrt{1+x^2}\dfrac{dy}{dx} = n\cdot ny$ (from (i))

$= n^2 y$ 1 m

9. $$\text{L H D at } x=1: \lim_{x\to 1^-}\left(\frac{x-1}{x-1}\right) = 1$$

$$\text{R H D at } x=1,\ \lim_{x\to 1^+}\frac{2-x-1}{x-1} = -1$$

2 m

$\therefore$ f is not differentiable at $x=1$

$$\text{L H D at } x=2,\ \lim_{x\to 2^-}\frac{2-x-0}{x-2} = -1$$

$$\text{R H D at } x=2,\ \lim_{x\to 2^+}\frac{-2+3x-x^2}{(x-2)} = \lim_{x\to 2^+} - \frac{(x-1)(x-2)}{(x-2)} = -1$$

2 m

$\therefore$ f is diff. at $x = 2$

10. $$\text{Communication Matrix } A = \begin{array}{c} \begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} \end{array} \begin{array}{l} \text{Telephone} \\ \text{House calls} \\ \text{Letters} \end{array}$$

$$\text{Cost Matrix } B = \begin{array}{cl} \begin{array}{ccc} \text{Tele} & \text{House calls} & \text{Letters} \end{array} & \\ \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} & \begin{array}{l} \text{City x} \\ \text{City y} \end{array} \end{array}$$

$$\therefore \text{ Total cost Matrix } = \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix}\begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} = \begin{pmatrix} 990000 \\ 2120000 \end{pmatrix}$$ 3 m

any relevant value 1 m

11. $$I = \int e^{2x} \sin(3x+1)\,dx = \left[\frac{e^{2x}}{2}\cdot \sin(3x+1) - \frac{3}{2}\int e^{2x}\cos(3x+1)\,dx\right]$$ 1½ m

$$= \frac{e^{2x}}{2}\cdot \sin(3x+1) - \frac{3}{2}\cdot\frac{e^{2x}}{2}\cdot \cos(3x+1) - \frac{9}{4}\int e^{2x}\sin(3x+1)\,dx \;\Big]$$ 1 m

$$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1) - \frac{9}{4} I$$ 1 m

$$\frac{13}{4} I = \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1)$$

$$I = \frac{4}{13}\left[\frac{e^{2x}}{2}\left(-\frac{3}{2}\cos(3x+1) + \sin(3x+1)\right)\right] + c$$ ½ m

12. Writing given integral as 1 m

$$I = \int_{-\pi/2}^{0} \frac{\cos x}{1+e^x} dx + \int_{0}^{\pi/2} \frac{\cos x}{1+e^x} dx$$ Let $x = -t$, $dx = -dt$ 1½ m

when $x = -\frac{\pi}{2}$, $t = \pi/2$

$x = 0$, $t = 0$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{e^t \cos t}{1+e^t} dt + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx = \int_0^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx$$ 1½ m

$$I = \int_0^{\frac{\pi}{2}} \frac{(1+e^x)\cos x \, dx}{(1+e^x)} = \int_0^{\frac{\pi}{2}} \cos x \, dx = (\sin x)_0^{\frac{\pi}{2}} = 1$$ 1 m

13. Let B_1, B_2, B_3 be the events that the bolts produced by machines E_1, E_2, E_3 and A be the event that the selected bulb is defective ½ m

$$\therefore P(B_1) = \frac{1}{2}, \; P(B_2) = P(B_3) = \frac{1}{4}$$

1½ m

$$P\left(A/B_1\right) = \frac{1}{25}, \; P\left(A/B_2\right) = \frac{1}{25}, \; P\left(A/B_3\right) = \frac{1}{20}$$

$$P(A) = \sum_{c=1}^{3} P(B_c) P\left(A/B_c\right) = \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20} + \frac{1}{4} \times \frac{1}{25}$$ 1+½ m

$$= \frac{17}{400}$$ ½ m

OR

$$\therefore\ P(x=3) = \frac{1}{6}\times\frac{1}{5}\times 2 = \frac{1}{15},\quad P(x=4) = \frac{2}{6}\times\frac{1}{5}\times 2 = \frac{2}{15}$$

Simlarly $P(x=5) = \frac{3}{15},\ P(x=6) = \frac{4}{15},\ P(x=7) = \frac{5}{15}$

Prob. distribution is

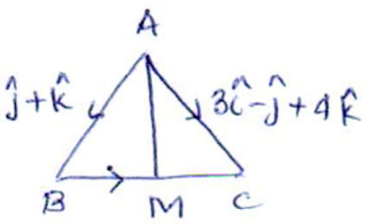
x :	3	4	5	6	7
P (x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$
x . P (x) :	$\frac{3}{15}$	$\frac{8}{15}$	$\frac{15}{15}$	$\frac{24}{15}$	$\frac{35}{15}$
x^2 P (x) :	$\frac{9}{15}$	$\frac{32}{15}$	$\frac{75}{15}$	$\frac{144}{15}$	$\frac{245}{15}$

2 m

$$\text{Mean} = \sum x_i \cdot P(x_i) = \frac{85}{15} = \frac{17}{3}$$ 1 m

$$\text{Variance} = \sum x_i^2\, P(x_i) - (\text{Mean})^2 = \frac{101}{3} - \frac{289}{9} = \frac{14}{9}$$ 1 m

14.



$$\vec{BC} = \left(3\hat{i} - \hat{j} + 4\hat{k}\right) - \left(\hat{j} + \hat{k}\right) = 3\hat{i} - 2\hat{j} + 3\hat{k}$$ 1½ m

$$\therefore\ \vec{BM} = \frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k}$$ 1 m

$$AM = \left|\hat{j} + \hat{k} + \frac{3\hat{i} - 2\hat{j} + 3\hat{k}}{2}\right| = \left|\frac{3\hat{i} + 5\hat{k}}{2}\right| = \frac{\sqrt{34}}{2}$$ 1½ m

15. Any plane through given point is $a(x-3)+b(y-6)+c(z-4)=0$..........(i) 1 m

with $a+5b+4c=0$ ....(A) ½ m

(i) passes through (3, 2, 0) $\Rightarrow -4b-4c=0$ or $b+c=0$ ....(B) ½ m

From (A) and (B) $a+b+(4b+4c) = 0 \Rightarrow a=-b$ 1 m

$\therefore\ a=-b=c$

$\therefore$ Required eqn. of plane is $x-y+z-1=0$ 1 m

16. LHS $= \tan^{-1}\left(\dfrac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}\dfrac{2\cos\theta}{\sin^2\theta}$ 2 m

$\therefore\ \tan^{-1}\dfrac{2\cos\theta}{\sin^2\theta} = \tan^{-1}\left(\dfrac{2}{\sin\theta}\right)$ 1 m

$\Rightarrow \cot\theta=1$ or $\theta = \pi/4$ 1 m

OR

The given equation can be written

$\left(\tan^{-1}2-\tan^{-1}1\right)+\left(\tan^{-1}3-\tan^{-1}2\right)+\left(\tan^{-1}4-\tan^{-1}3\right)+.....+\tan^{-1}(n+1)-\tan^{-1}n=\tan^{-1}\theta$ 2 m

$\Rightarrow\ \tan^{-1}(n+1)-\tan^{-1}1 = \tan^{-1}\theta$ 1 m

$\Rightarrow\ \tan^{-1}\dfrac{n+1-1}{1+(n+1)} = \tan^{-1}\theta \Rightarrow \theta = \dfrac{n}{n+2}$ 1 m

17. Getting $A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$ 1 m

$4A - 3I = \begin{bmatrix} 8-3 & -4 \\ -4 & 8-3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} = A^2$ 1 m

$A^2 = 4A - 3I$ ....(i)

Multiply both sides by A^{-1} ½ m

$A = 4I - 3A^{-1}$ or $A^{-1} = \frac{1}{3}(4I - A) = \frac{1}{3}\begin{pmatrix} 4-2 & 0+1 \\ 0+1 & 4-2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ 1½ m

OR

$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a^2+b & a-1 \\ b(a-1) & b+1 \end{bmatrix}$ 1½ m

$(A+B)^2 = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} (1+a)^2 & 0 \\ (2+b)(1+a) - 2(2+b) & 4 \end{pmatrix}$ .... (i) 1½ m

$A^2 + B^2 = \begin{pmatrix} a^2+b-1 & a-1 \\ b(a-1) & b \end{pmatrix}$ ....(ii)

Equating (i) and (ii), we get $b = 4, a = 1$ 1 m

18. Using $C_1 \to C_1 + C_2 + C_3$ and taking $a^2 + a + 1$ common from C_1

$\Delta = (a^2 + a + 1)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix}$, using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ 1½ m

$$\Delta = (a^2+a+1)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1-a & a(1-a) \\ 0 & a(a-1) & (1-a)(1+a) \end{vmatrix} = (a^2+a+1)(1-a)^2\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 0 & -a & 1+a \end{vmatrix}$$ 1½ m

$$\left.\begin{aligned} &= (a^2+a+1)(1-a)^2(1+a+a^2) \\ &= [(1-a)(1+a+a^2)]^2 = (1-a^3)^2 \end{aligned}\right\}$$ 1 m

19. Let $x+a=t \Rightarrow dx = dt$ and $x = t-a \Rightarrow x-a = t-2a$ 1 m

$$\therefore\ I = \int \frac{\sin(t-2a)\,dt}{\sin t} = \int \frac{(\sin t \cos 2a - \cos t \cdot \sin 2a)\,dt}{\sin t}$$ 1 m

$$= \cos 2a \int dt - \sin 2a \int \cot t\, dt = \cos 2a\ t - \sin 2a\,.\log|\sin t| + c$$ 1 m

$$= \cos 2a\,(x+a) - \sin 2a \log|\sin(x+a)| + c$$ 1

OR

consider $\dfrac{x^2}{(x^2+4)(x^2+9)}$ · Let $x^2 = t$ ½ m

$$\therefore\ \frac{t}{(t+4)(t+9)} = -\frac{4}{5}\,\frac{1}{t+4} + \frac{9}{5}\,\frac{1}{t+9}$$ 1 m

$$I = \int \frac{t \quad dt}{(t+4)(t+9)} = -\frac{4}{5}\int\frac{dt}{t+4} + \frac{9}{5}\int\frac{dt}{t+9}$$ ½ m

$$= \frac{-4}{5}\log|t+4| + \frac{9}{5}\log|t+9| + c$$ 1½ m

$$\therefore\ I = -\frac{4}{5}\log|x^2+4| + \frac{9}{5}\log|x^2+9| + c$$ ½ m

SECTION - C

20. $$\frac{dy}{dx} = \frac{2x\sin\left(\frac{y}{x}\right) - y\cos\left(\frac{y}{x}\right)}{y - x\cos\left(\frac{y}{x}\right)} = \frac{2\sin\left(\frac{y}{x}\right) - \frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{y}{x} - \cos\left(\frac{y}{x}\right)}$$ 1 m

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 m

$\therefore\ v + x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v - \cos v} \Rightarrow x\frac{dv}{dx} = \frac{2\sin v - v^2}{v - \cos v}$ 1 m

$\Rightarrow \frac{v - \cos v}{-2\sin v + v^2} = \frac{-dx}{x}$ or $\frac{1}{2}\left[\frac{2v - 2\cos v}{-2\sin v + v^2}\right]dv = -\frac{dx}{x}$ 1 m

$\Rightarrow \frac{1}{2}\log\left|v^2 - 2\sin v\right| = -\log x + \log c$

or $\log\left|\sqrt{v^2 - 2\sin v}\right| = \log c - \log x$ 1 m

$\sqrt{v^2 - 2\sin v} = \frac{c}{x}$

or $x\sqrt{\frac{y^2}{x^2} - 2\sin\frac{y}{x}} = c$ ½ m

$y^2 - 2x^2\sin\left(\frac{y}{x}\right) = c'$ ½ m

OR

$\left(\sqrt{(1+x^2)(1+y^2)}\right)dx + xy\,dy = 0$

$\Rightarrow \frac{y}{\sqrt{1+y^2}}dy + \frac{\sqrt{1+x^2}}{x}dx = 0$ 1 m

$\frac{1}{2}\int\frac{2y}{\sqrt{1+y^2}}dy + \int\frac{\sqrt{1+x^2}}{x}dx = 0$

$\sqrt{1+y^2} + \int\frac{(1+x^2)}{x\sqrt{1+x^2}}dx = c$ 1½ m

$$I_1 = \int \frac{1}{x\sqrt{1+x^2}}\ dx + \int \frac{x}{\sqrt{1+x^2}}\ dx = I_2 + \sqrt{1+x^2}$$ 1 m

For I_2, Let $x = \frac{1}{t}$, $dx = \frac{-1}{t^2} dt$ 1 m

$$I_2 = \int \frac{-1}{t^2 \cdot \frac{1}{t}\sqrt{1+\frac{1}{t^2}}}\ dt = -\int \frac{dt}{\sqrt{t^2+1}} = -\log\left[t+\sqrt{1+t^2}\right]$$ 1 m

$$= -\log\left[\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right] = -\log\left[\frac{1+\sqrt{1+x^2}}{x}\right]$$

$\therefore$ The solution is $\sqrt{1+x^2}+\sqrt{1+y^2} - \log\left(\frac{1+\sqrt{1+x^2}}{2}\right) = c$ ½ m

21. $P(\text{Doublet}) = \frac{1}{6}$, $P(\text{not a doublet}) = 5/6$

The random variate x can take values 0, 1, 2, 3, 4 1 m

x	0	1	2	3	4	
P(x)	$\left(\frac{5}{6}\right)^4$	$4\ \frac{1}{6}\left(\frac{5}{6}\right)^3$	$6\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$	$4\left(\frac{1}{6}\right)^3\frac{5}{6}$	$\left(\frac{1}{6}\right)^4$	
	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$	2½ m

$$\text{Mean} = \sum x\ P(x) = \frac{500+300+60+4}{1296} = \frac{864}{1296} = \frac{2}{3}$$ 1 m

$$\sum x^2\ P(x) = \frac{500+600+180+16}{1296} = \frac{1296}{1296} = 1$$ 1 m

$$\therefore \text{ Variance } = 1-\left(\frac{2}{3}\right)^2 = \frac{5}{9}$$ ½ m

22. Let $x_1, x_2, \in R$ such that $f(x_1) = f(x_2) \Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$ 1½+½ m

$\Rightarrow 4(x_1 - x_2)[x_1 + x_2 + 3] = 0 \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one – one

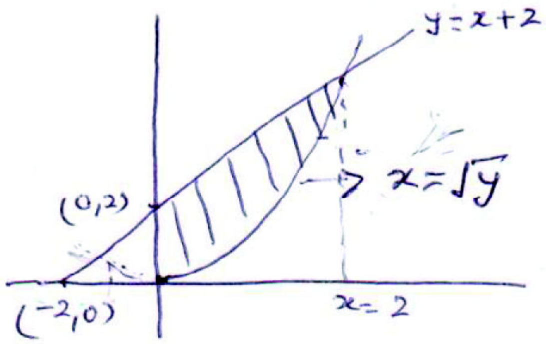
f is clearly onto and hence intertible 1 m

Let y be an arbitrary element of S

$f(x) = y = 4x^2 + 12x + 15 = (2x+3)^2 + 6$ 1 m

$\therefore\ f^{-1}: R \to S$ is given by $f^{-1}(y) = \left(\dfrac{\sqrt{y-6}-3}{2}\right)$ 2 m

23.



Correct Figure 1m

Points of intersection

$x^2 - x - 2 = 0$ 1½ m

$(x-2)(x+1) = 0$

$x = 2, -1$ (-1 is rejected)

$\therefore$ Reqd. area $= \displaystyle\int_0^2 \{(x+2) - x^2\}dx$ 1½ m

$= \left[\dfrac{x^2}{2} + 2x - \dfrac{x^3}{3}\right]_0^2$

$= \left(2 + 4 - \dfrac{8}{3}\right) = \dfrac{10}{3}$ sq.units 2 m

24.

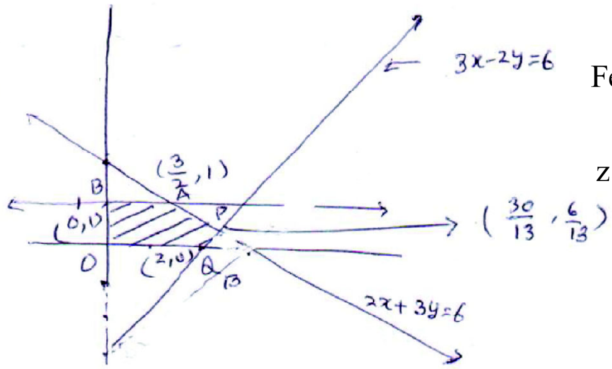


Figure 3 m

Feasible region is B A PQO 1 m

$$\left.\begin{aligned} z_B = 9,\ z_P &= \frac{240}{13} + \frac{54}{13} = \frac{294}{13} \\ &= 22\,\frac{8}{13} \end{aligned}\right\}$$ 1 m

$Z_Q = 16$

$$\left.\begin{aligned} &\therefore\ Z \text{ is maximum at } \left(\frac{30}{13}, \frac{6}{13}\right) \\ &\text{and maximum value} = 22\,\frac{8}{13} \end{aligned}\right\}$$ 1 m

25. Any line through $(1,-2,3)$ with d. r's as 2, 3 – 6 is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$$ 1 ½ m

$\therefore\ x = 2\lambda+1,\ y = 3\lambda-2,\ z = -6\lambda+3$ 1½ m

It lies on the plane $x-y+z=5$

$\therefore\ 2\lambda+1-3\lambda+2-6\lambda+3=5$

$\Rightarrow\ \lambda = \frac{1}{7}$ 1 m

Reqd. point is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$ 1 m

$\therefore$ Reqd distance $= \sqrt{\left(\frac{9}{7}-1\right)^2+\left(-\frac{11}{7}+2\right)^2+\left(\frac{15}{7}-3\right)^2} = \frac{1}{7}\sqrt{4+9+36} = \frac{7}{7} = 1$ 1 m

26. Let $z = ax+by$, also $xy = c^2 \Rightarrow y = \frac{c^2}{x}$ ½ + ½ m

$\therefore\ z = ax + \frac{bc^2}{x}$

$\therefore\ \frac{dz}{dx} = a + bc^2\left(\frac{-1}{x^2}\right),\ \frac{dz}{dx} = 0 \Rightarrow bc^2 = ax^2$

or $x = \sqrt{\frac{b}{a}}\,c$ 1½ m

showing $\frac{d^2z}{ax^2}$ at $x = \sqrt{\frac{b}{a}}\,c > 0 \Rightarrow$ minima 1½ m

$$y = \frac{c^2}{x} = \frac{c^2}{c}\sqrt{\frac{a}{b}} = c\sqrt{\frac{a}{b}}$$ 1 m

$$\therefore \text{ minimum } z = a\sqrt{\frac{b}{a}}\,c + bc\sqrt{\frac{a}{b}}\,c = 2\sqrt{ab}\,c$$ 1 m

OR

$y = x^2 + 7x + 2\ ,\ 3x - y - 3 = 0$ ....(i)

$\therefore\ 3x - \left(x^2 + 7x + 2\right) - 3 = 0$ 1 m

Distance of (x, y) from (i)

$$D = \left| \frac{3x - \left(x^2 + 7x + 2\right) - 3}{\sqrt{10}} \right| \quad \text{or} \quad D = \left| \frac{\left(-x^2 - 4x - 5\right)}{\sqrt{10}} \right| = \left| \frac{(x+2)^2 + 1}{\sqrt{10}} \right|$$ 2 m

$$\frac{dD}{dx} = \frac{2}{\sqrt{10}}\left(x+2\right),\ \frac{dD}{dx} = 0 \text{ at } x = -2$$ 1 m

$$\frac{d^2D}{dx^2} > 0 \Rightarrow \text{minima}$$ 1 m

$\therefore$ D is minimum at $x = -2$

at $x = -2,\ y = -8$ 1 m

$\therefore$ The required pt. on the parabola is $\left(-2, -8\right)$