SET – 4

Series : GBM/C

कोड नं.
Code No. 65(B)

रोल नं.
Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **12** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **29** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **12** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

(केवल नेत्रहीन परीक्षार्थियों के लिए)

MATHEMATICS

(FOR BLIND CANDIDATES ONLY)

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 100*

Time allowed : 3 hours | *Maximum Marks : 100*

सामान्य निर्देश :

(i) *सभी प्रश्न अनिवार्य हैं ।*

(ii) *इस प्रश्न–पत्र में* ***29*** *प्रश्न हैं ।*

(iii) *खण्ड–अ में प्रश्न* ***1-4*** *अति लघुउत्तर प्रकार के प्रश्न हैं, जिनमें प्रत्येक* ***1*** *अंक का है ।*

(iv) *खण्ड–ब में प्रश्न* ***5-12*** *लघुउत्तर प्रकार के प्रश्न हैं, जिनमें प्रत्येक* ***2*** *अंक का है ।*

(v) *खण्ड–स में प्रश्न* ***13-23*** *दीर्घ उत्तर–**I** प्रकार के प्रश्न हैं, जिनमें प्रत्येक* ***4*** *अंक का है ।*

(vi) *खण्ड–द में प्रश्न* ***24-29*** *दीर्घ उत्तर–**II** प्रकार के प्रश्न हैं, जिनमें प्रत्येक* ***6*** *अंक का है ।*

General Instructions :

(i) ***All*** *questions are compulsory.*

(ii) *This question paper contains* ***29*** *questions.*

(iii) *Question* ***1-4*** *in Section A are very short-answer type questions carrying* ***1*** *mark each.*

(iv) *Question* ***5-12*** *in Section B are short answer type questions carrying* ***2*** *marks each.*

(v) *Question* ***13-23*** *in Section C are long-answer – I type questions carrying* ***4*** *marks each.*

(vi) *Question* ***24-29*** *in Section D are long-answer – II type questions carrying* ***6*** *marks each.*

खण्ड – अ

SECTION – A

प्रश्न संख्या 1 से 4 तक प्रत्येक प्रश्न 1 अंक का है ।

Question numbers **1** to **4** carry **1** mark each.

1. यदि $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$ है तो $(A + A')$ ज्ञात कीजिए ।

   If $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$, then find $(A + A')$.

2. $x \log x$ का x के सापेक्ष अवकलन कीजिए ।

   Differentiate $x \log x$ w.r.t. x.

3. $\int_0^{\pi} \cos^5 x \, dx$ का मान लिखिए ।

   Write the value of $\int_0^{\pi} \cos^5 x \, dx$.

4. यदि एक रेखा AB के समीकरण $\frac{3-x}{-1} = \frac{y+2}{2} = \frac{2z-5}{4}$ हैं, तो रेखा AB के समांतर रेखा के दिक् अनुपात ज्ञात कीजिए ।

   If the equations of a line AB are $\frac{3-x}{-1} = \frac{y+2}{2} = \frac{2z-5}{4}$, then find the direction ratios of a line parallel to AB.

खण्ड – ब

SECTION – B

प्रश्न संख्या **5** से **12** तक प्रत्येक प्रश्न **2** अंक का है ।
Question numbers **5** to **12** carry **2** marks each.

5. किसी आयत की लंबाई x, 3 सेमी/मिनट की दर से घट रही है और उसकी चौड़ाई y, 2 सेमी/मिनट की दर से बढ़ रही है । जब $x = 10$ सेमी तथा $y = 6$ सेमी है, तो आयत के क्षेत्रफल में परिवर्तन की दर ज्ञात कीजिए ।

   The length x of a rectangle is decreasing at the rate of 3 cm/minute while its breadth y is increasing at the rate of 2 cm/min. When $x = 10$ cm and $y = 6$ cm, find the rate of change of area of the rectangle.

6. $\tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)$ का x के सापेक्ष अवकलन कीजिए ।

   Find the derivative of $\tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)$ w.r.t. x.

7. दर्शाइए कि $A = \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}$ आव्यूह समीकरण $A^2 - 3A - 7I = O$ को संतुष्ट करती है । अतः A^{-1} ज्ञात कीजिए ।

   Show that $A = \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}$ satisfies the matrix equation $A^2 - 3A - 7I = O$, hence find A^{-1}.

8. उस रेखा का कार्तीय तथा सदिश समीकरण ज्ञात कीजिए जो बिंदु (3, –7, –4) से होकर जाती है तथा रेखा $\frac{x}{2} = \frac{y}{-1} = \frac{z+1}{3}$ के समांतर है ।

Find the Cartesian and Vector equation of a line passing through the point (3, –7, –4) and parallel to the line $\frac{x}{2} = \frac{y}{-1} = \frac{z+1}{3}$.

9. वह अंतराल ज्ञात कीजिए जिनमें फलन $f(x) = 4x^3 - 6x^2 - 72x + 30$

(i) निरंतर वर्धमान है ।

(ii) निरंतर ह्रासमान है ।

Find the intervals in which the function $f(x) = 4x^3 - 6x^2 - 72x + 30$ is

(i) strictly increasing

(ii) strictly decreasing

10. ज्ञात कीजिए : $\int \frac{3}{\sqrt{5 - 4x - x^2}}\, dx$

Find : $\int \frac{3}{\sqrt{5 - 4x - x^2}}\, dx$

11. एक व्यक्ति ₹ 75,000 की राशि तक निवेश करना चाहता है । उसके लिए दो प्रकार के बाँड B_1 तथा B_2 उपलब्ध हैं । बाँड B_1 8% ब्याज देता है जबकि बाँड B_2 9% ब्याज देता है । वह निश्चय करता है बाँड B_1 में कम से कम ₹ 20,000 निवेश करे तथा बाँड B_2 में ₹ 35,000 से अधिक नहीं । वह यह भी चाहता है कि बाँड B_1 में कम से कम बाँड B_2 जितनी राशि निवेश करे । अधिकतम ब्याज पाने के लिए इसे रैखिक प्रोग्रामन समस्या बनाकर सूत्रबद्ध कीजिए ।

A person wants to invest upto ₹ 75,000. For this two types of Bonds B_1 and B_2 are available. Bond B_1 gives 8% interest while Bond B_2 yields 9% interest. He decides to invest at least ₹ 20,000 in Bond B_1 and not more than ₹ 35,000 in Bond B_2. He also wants to invest at least as much in Bond B_1 as in Bond B_2. Make it an LPP for maximising the interest and formulate the problem.

12. यदि A तथा B दो स्वतंत्र घटनाएँ हैं तथा $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ है, तो $P(A \cup B)$ ज्ञात कीजिए । अतः P(A नहीं तथा B नहीं) का मान ज्ञात कीजिए ।

If A and B are two independent events and $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, find $P(A \cup B)$. Hence find P(not A and not B).

खण्ड – स

SECTION – C

प्रश्न संख्या **13** से **23** तक प्रत्येक प्रश्न **4** अंकों का है ।

Question numbers **13** to **23** carry **4** marks each.

13. ज्ञात कीजिए : $\int \frac{x^2}{(x^2+1)(x^2+4)} \, dx$

Find : $\int \frac{x^2}{(x^2+1)(x^2+4)} \, dx$

14. सारणिकों के गुणधर्मों का प्रयोग कर निम्न सिद्ध कीजिए :

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

अथवा

आव्यूह A ज्ञात कीजिए जो निम्न आव्यूह समीकरण को संतुष्ट करता है :

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} A \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Using properties of determinants, prove the following :

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

OR

Find matrix A such that it satisfies the following matrix equation :

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} A \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

15. x के लिए हल कीजिए : $\cot^{-1}x - \cot^{-1}(x+2) = \frac{\pi}{4}, x > 0$

अथवा

दर्शाइए कि : $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

Solve for x : $\cot^{-1}x - \cot^{-1}(x+2) = \frac{\pi}{4}, x > 0$

OR

Show that : $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

16. $(x \cos x)^x + (\sin x)^{\cos x}$ का अवकलन x के सापेक्ष ज्ञात कीजिए ।

अथवा

यदि $y = a(\sin t - t \cos t)$ और $x = a(\cos t + t \sin t)$ है, तो $t = \frac{\pi}{4}$ पर $\frac{d^2y}{dx^2}$ का मान ज्ञात कीजिए ।

Find the derivative of $(x \cos x)^x + (\sin x)^{\cos x}$ w.r.t. x.

OR

If $y = a(\sin t - t \cos t)$ and $x = a(\cos t + t \sin t)$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

17. अवकल समीकरण $2ye^{\frac{x}{y}}dx + (y - 2xe^{\frac{x}{y}})\ dy = 0$ का विशिष्ट हल ज्ञात कीजिए, दिया है कि $y = 1$, जब $x = 0$.

Find the particular solution of the differential equation $2ye^{\frac{x}{y}}dx + (y - 2xe^{\frac{x}{y}})dy = 0$, given that $y = 1$, when $x = 0$.

18. मान ज्ञात कीजिए : $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$

Evaluate : $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$

19. माना $\vec{a}$ तथा $\vec{b}$ ऐसे सदिश हैं कि $|\vec{a}| = 3$, $|\vec{b}| = \frac{\sqrt{2}}{3}$ यदि $\vec{a} \times \vec{b}$ एक इकाई सदिश है, तो $\vec{a}$ तथा $\vec{b}$ के बीच का कोण ज्ञात कीजिए।

Let $\vec{a}$ and $\vec{b}$ be such vectors that $|\vec{a}| = 3$, $|\vec{b}| = \frac{\sqrt{2}}{3}$. If $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

20. एक ऐसा सदिश ज्ञात कीजिए जिसका परिमाण 3 इकाई है तथा वह सदिशों $\vec{a}$ तथा $\vec{b}$ पर लंबवत है जहाँ $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ तथा $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$ है।

Find a vector whose magnitude is 3 units and which is perpendicular to the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$.

21. दो दर्जी A तथा B को काम के लिए प्रतिदिन क्रमशः ₹ 225 तथा ₹ 300 मिलते हैं। A प्रतिदिन 9 कमीजें तथा 6 पेंट सिल सकता है जबकि B प्रतिदिन 15 कमीजें तथा 6 पेंट सिल सकता है। कम से कम लागत में 90 कमीजें तथा 48 पेंट बनाने के लिए उपरोक्त को रैखिक प्रोग्रामन समस्या बनाकर सूत्रबद्ध कीजिए।

यदि दोनों दर्जी किन्हीं विकलांगों की संस्था द्वारा ऑर्डर देने पर प्रतिदिन की आमदनी से 25% कम पर काम करने को तैयार हों, तो उनके द्वारा क्या मूल्य प्रदर्शित होता है।

Two tailors A and B are paid ₹ 225 and ₹ 300 per day respectively for work. A can stitch 9 shirts and 6 pants per day while B can stitch 15 shirts and 6 pants per day. Formulate the above linear programming problem for minimum cost to stitch 90 shirts and 48 pants.

If both the tailors agree to charge 25% less daily on an order by a handicapped institute, what value do they demonstrate.

22. पासों के एक जोड़े को तीन बार उछालने पर आने वाले द्विकों (doublets) की संख्या का प्रायिकता बंटन ज्ञात कीजिए। अतः इस बंटन का माध्य ज्ञात कीजिए।

Find the probability distribution of number of doublets in three throws of a pair of dice. Hence find the mean of the distribution.

23. एक बोल्ट बनाने के कारखाने में मशीनें A, B तथा C कुल उत्पादन का क्रमशः 25%, 35% तथा 40% बोल्ट बनाती हैं। इन मशीनों के उत्पादन का क्रमशः 5%, 4% तथा 2% भाग त्रुटिपूर्ण है। बोल्टों के कुल उत्पादन में से एक बोल्ट यादृच्छया निकाला जाता है और त्रुटिपूर्ण पाया जाता है। प्रायिकता ज्ञात कीजिए कि यह बोल्ट मशीन B द्वारा बनाया गया है।

In a factory, manufacturing bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their output 5%, 4% and 2% respectively are found to be defective bolts. A bolt is drawn at random from the total production and is found to be defective. Find the probability that it is manufactured by machine B.

खण्ड – द

SECTION – D

प्रश्न संख्या **24** से **29** तक प्रत्येक प्रश्न **6** अंक का है ।

Question numbers **24** to **29** carry **6** marks each.

24. आव्यूहों $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$ का गुणनफल ज्ञात कीजिए । इसका प्रयोग कर निम्न समीकरण निकाय को हल कीजिए :

$x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$

अथवा

आव्यूह $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$ के लिए दर्शाइए कि $A^3 - 6A^2 + 5A + 11I = O$.

Find the product of the matrices $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$ and use it to solve the system of equations :

$x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$

OR

For the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$, show that $A^3 - 6A^2 + 5A + 11I = O$.

25. माना $A = R - \{1\}$ यदि $f : A \to A$ एक फलन है जो $f(x) = \dfrac{x-2}{x-1}$ द्वारा परिभाषित है तो दर्शाइए कि f एकैकी तथा आच्छादक है । अतः f^{-1} ज्ञात कीजिए ।

निम्न भी ज्ञात कीजिए :

(i) x, जब $f^{-1}(x) = \dfrac{5}{6}$

(ii) $f^{-1}(2)$

Let $A = R - \{1\}$. If $f : A \to A$ is a mapping defined by $f(x) = \dfrac{x-2}{x-1}$, show that f is bijective, find f^{-1}. Also find :

(i) x if $f^{-1}(x) = \dfrac{5}{6}$

(ii) $f^{-1}(2)$

26. सिद्ध कीजिए कि एक दिए गए वृत्त के अंतर्गत बने सभी आयतों में से वर्ग का क्षेत्रफल अधिकतम होता है ।

Show that of all the rectangles inscribed in a given circle, the square has the maximum area.

27. उस समतल का कार्तीय तथा सदिश समीकरण ज्ञात कीजिए जो बिन्दु $(-1, 3, 2)$ से होकर जाता है तथा समतलों $x + 2y + 3z = 5$ तथा $3x + 3y + z = 0$ दोनों पर लंबवत है । अतः दर्शाइए कि इस प्रकार प्राप्त समतल रेखा $\dfrac{x+1}{5} = \dfrac{y-4}{4} = \dfrac{z+1}{-1}$ के समांतर है ।

अथवा

उस बिंदु के निर्देशांक ज्ञात कीजिए जहाँ बिंदुओं $(3, -4, -5)$ तथा $(2, -3, 1)$ से होकर जाने वाली रेखा, बिंदुओं $(1, 1, 4)$, $(3, -1, 2)$ तथा $(4, 1, -2)$ द्वारा निर्धारित समतल को काटती है ।

Find the Cartesian and Vector equations of the plane passing through the point (–1, 3, 2) and is perpendicular to each of the planes :

$x + 2y + 3z = 5$, $3x + 3y + z = 0$. Hence show that the line $\frac{x+1}{5} = \frac{y-4}{4} = \frac{z+1}{-1}$ is parallel to the plane thus obtained.

OR

Find the co-ordinates of the point where the line through (3, –4, –5) and (2, –3, 1) crosses the plane determined by the points (1, 1, 4), (3, –1, 2) and (4, 1, –2).

28. अवकल समीकरण $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x, (x \neq 0)$ का एक विशिष्ट हल ज्ञात कीजिए, दिया है कि जब $x = \frac{\pi}{2}$ है तो $y = 0$ है ।

Find a particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x, (x \neq 0)$, given that $y = 0$ when $x = \frac{\pi}{2}$.

29. समाकलनों के प्रयोग से उस त्रिभुज द्वारा घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए जिसके शीर्ष (–1, 0), (1, 3) तथा (3, 2) हैं ।

अथवा

योगफल की सीमा के रूप में $\int_1^3 (3x^2 + e^{2x})\, dx$ का मान ज्ञान कीजिए ।

Using integration, find the area of the region bounded by the triangle whose vertices are (–1, 0), (1, 3) and (3, 2).

OR

Find $\int_1^3 (3x^2 + e^{2x})\, dx$ as limit of a sum.

Strictly Confidential — (For Internal and Restricted Use Only)

Senior Secondary School Certificate Examination

July 2017 (Compartment)

Marking Scheme — Mathematics 65(B)

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65(B)

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$ $\frac{1}{2}$

$A + A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$ $\frac{1}{2}$

2. $y = x \log x$

$\Rightarrow \quad \frac{dy}{dx} = 1 + \log x$ 1

3. $\int_0^{\pi} \cos^5 x \, dx = 0$ 1

4. AB: $\frac{x-3}{1} = \frac{y+2}{2} = \frac{z-5/2}{2}$

DR's of required line $< 1, 2, 2 >$ 1

SECTION B

5. $\frac{dx}{dt} = -3$ cm/min, $\frac{dy}{dt} = 2$ cm/min $\frac{1}{2}$

$A = x.y$

$\frac{dA}{dt} = x \cdot \frac{dy}{dt} + y \cdot \frac{dx}{dt}$ 1

$= 2\ cm^2/min$ $\frac{1}{2}$

i.e. Area is increasing at the rate of 2 cm^2/min.

6. $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right] \qquad \frac{1}{2}$$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right] \qquad \frac{1}{2}$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) \qquad \frac{1}{2}$$

$$= \frac{\pi}{4} - \frac{x}{2}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{2} \qquad \frac{1}{2}$$

7. $A^2 - 3A - 7I = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = O$ 1

Pre-multiplying (or Post multiplying) by A^{-1}, we get

$A^{-1} = \frac{1}{7}(A - 3I) = \begin{bmatrix} 2/7 & 3/7 \\ -1/7 & -5/7 \end{bmatrix}$ 1

8. Cartesian equation of required line is

$$\frac{x-3}{2} = \frac{y+7}{-1} = \frac{z+4}{3} \qquad 1$$

Vector equation of required line

$$\vec{r} = (3\hat{i} - 7\hat{j} - 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k}) \qquad 1$$

9. $f(x) = 4x^3 - 6x^2 - 72x + 30$

$f'(x) = 12x^2 - 12x - 72$ $\frac{1}{2}$

$f'(x) = 0 \Rightarrow x^2 - x - 6 = 0$

$\Rightarrow \quad (x-3)(x+2) = 0$

$\Rightarrow \quad x = -2 \text{ or } x = 3$ $\frac{1}{2}$

Disjoint intervals are $(-\infty, -2)$, $(-2, 3)$ and $(3, \infty)$ $\frac{1}{2}$

f(x) is strictly increasing on $(-\infty, -2), \cup (3, \infty)$

f(x) is strictly decreasing on $(-2, 3)$ $\frac{1}{2}$

10. $I = 3\int \frac{dx}{\sqrt{5 - 4x - x^2}}$

$= 3\int \frac{dx}{\sqrt{(3)^2 - (x+2)^2}}$ 1

$= 3\sin^{-1}\frac{x+2}{3} + C$ 1

11. Let amount invested in bond B_1 is Rs.x and in bond B_2 is Rs. y

L.P.P. is Maximum $Z = \frac{8}{100}x + \frac{9y}{100}$ $\frac{1}{2}$

subject to

$$\left.\begin{aligned} & x \geq 20000 \\ & y \leq 35000 \\ & x + y \leq 75000 \\ & x \geq y \\ & x, y \geq 0 \end{aligned}\right\} \qquad 1\frac{1}{2}$$

12. A and B are independent events

$\therefore \quad P(A \cap B) = P(A).P(B) = \frac{1}{8}$ $\frac{1}{2}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$ 1

$P(A' \cap B') = 1 - P(A \cup B) = 1 - \frac{5}{8} = \frac{3}{8}$ $\frac{1}{2}$

SECTION C

13. $x^2 = y$ (say) 1

$$\frac{y}{(y+1)(y+4)} = \frac{A}{y+1} + \frac{B}{y+4}$$ 1

Solving we get, $A = -\frac{1}{3},\ B = \frac{4}{3}$

$$\therefore \quad \int \frac{x^2}{(x^2+1)(x^2+4)} dx = -\frac{1}{3}\int \frac{dx}{(x^2+1)} + \frac{4}{3}\int \frac{dx}{x^2+4}$$

$$= -\frac{1}{3}\tan^{-1} x + \frac{2}{3}\tan^{-1}\frac{x}{2} + C$$ 2

14. $R_1 \to R_1 - R_2 - R_3$ 1

$$\begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$ 1

$= 2c[ab + b^2 - bc] - 2b[bc - c^2 - ac]$

$= 4abc$ 2

OR

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ 1

$$\Rightarrow \quad A\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$ 1

$$\Rightarrow \quad A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$ 1

$$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$ 1

15. $\cot^{-1} x - \cot^{-1}(x+2) = \dfrac{\pi}{4}, x > 0$

$$\Rightarrow \quad \tan^{-1}\frac{1}{x} - \tan^{-1}\frac{1}{x+2} = \frac{\pi}{4}$$ 1

$$\Rightarrow \quad \tan^{-1}\left[\frac{\frac{1}{x} - \frac{1}{x+2}}{1 + \frac{1}{x(x+2)}}\right] = \frac{\pi}{4}$$ 1

$$\Rightarrow \quad \frac{2}{x^2 + 2x + 1} = 1 \Rightarrow x^2 + 2x - 1 = 0$$ 1

$$\Rightarrow \quad x = \sqrt{2} - 1$$ 1

OR

$$\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5}$$

$$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4}$$ 1

$$= \tan^{-1}\left[\frac{\frac{5}{12} + \frac{3}{4}}{1 - \frac{5}{16}}\right]$$ 1

$$= \tan^{-1}\frac{56}{33} = \sin^{-1}\frac{56}{65}$$ 1+1

16. $y = (x\cos x)^x + (\sin x)^{\cos x}$

Let $u = (x\cos x)^x$

$\Rightarrow \quad \log u = x(\log x + \log\cos x)$ $\frac{1}{2}$

$\Rightarrow \quad \frac{1}{u}\cdot\frac{du}{dx} = x\left[\frac{1}{x} - \tan x\right] + \log(x\cos x)$

$\Rightarrow \quad \frac{du}{dx} = (x\cos x)^x\,[1 - x\tan x + \log(x\cos x)] \qquad ...(i)$ 1

$v = (\sin x)^{\cos x}$

$\log v = \cos x \log\sin x$ $\frac{1}{2}$

$\Rightarrow \quad \frac{1}{v}\cdot\frac{dv}{dx} = \cos x\,.\,\cot x + \log\sin x\,.\,(-\sin x)$

$\Rightarrow \quad \frac{dv}{dx} = (\sin x)^{\cos x}\left[\frac{\cos^2 x}{\sin x} - \sin x\log\sin x\right] \qquad ...(ii)$ 1

$\therefore \quad \frac{dy}{dx} = (x\cos x)^x[1 - x\tan x + \log(x\cos x)] + (\sin x)^{\cos x}\left[\frac{\cos^2 x}{\sin x} - \sin x\log\sin x\right]$ 1

OR

$y = a(\sin t - t\cos t)$

$\frac{dy}{dt} = a[\cos t + t\sin t - \cos t]$ 1

$= a\,t\sin t$

$x = a[\cos t + t\sin t]$

$\frac{dx}{dt} = a[-\sin t + t\cos t + \sin t]$ 1

$= a\,t\cos t$

$\therefore \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t$ $\frac{1}{2}$

$$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx}$$

$$= \frac{\sec^2 t}{a\,t \cos t} = \frac{1}{a\,t}\sec^3 t$$ 1

$$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{\pi}{4}} = \frac{8\sqrt{2}}{\pi a}$$ $\frac{1}{2}$

17. Given differential equation can be written as

$$\frac{dx}{dy} = \frac{2x\,e^{x/y} - y}{2y\,e^{x/y}}$$ $\frac{1}{2}$

Put $x = vy$

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$ 1

$$v + y \cdot \frac{dv}{dy} = v - \frac{1}{2e^v}$$

$$\int e^v\,dv = -\int \frac{dy}{2y}$$ 1

$$e^v = -\frac{1}{2}\log|y| + C$$

$\Rightarrow$ $$e^{x/y} = -\frac{1}{2}\log|y| + C$$ 1

when $x = 0$, $y = 1$, we get $C = 1$

$\therefore$ $$e^{x/y} = 1 - \frac{1}{2}\log|y|$$ $\frac{1}{2}$

18. $$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1 + \cos^2(\pi - x)}dx$$ 1

$\Rightarrow$ $$2I = \pi\int_0^{\pi} \frac{\sin x}{1 + \cos^2 x}dx$$ 1

$$= 2\pi\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$$

$$= -2\pi\int_1^0 \frac{dt}{1+t^2}$$ 1

put $\cos x = t, -\sin x\, dx = dt$

$$= 2\pi[\tan^{-1} t]_0^1$$

$$= \frac{\pi^2}{2}$$

$$\Rightarrow \quad I = \frac{\pi^2}{4}$$ 1

19. $|\vec{a} \times \vec{b}| = 1$ 1

$\Rightarrow \quad |\vec{a}||\vec{b}|\sin\theta = 1$ 1

$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}}$ 1

$\Rightarrow \quad \theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ 1

20. Unit vector perpendiculare to $\vec{a}$ and $\vec{b}$

$$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$ 1

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -4 \\ 6 & 5 & -2 \end{vmatrix} = 18\hat{i} - 18\hat{j} + 9\hat{k}$$ 1

$$|\vec{a}\times\vec{b}| = 27$$

$$\therefore \quad \hat{n} = \frac{2\hat{i} - 2\hat{j} + \hat{k}}{3}$$ 1

Required vector $= 2\hat{i} - 2\hat{j} + \hat{k}$ 1

21. Let A worked for x days and B worked for y days

Minimise $z = 225x + 300y$ 1

subject to constraints

$$\left.\begin{array}{l} 9x + 15y \geq 90 \Rightarrow 3x + 5y \geq 30 \\ 6x + 6y \geq 48 \Rightarrow x + y \geq 8 \end{array}\right\}$$ 2

$x, y \geq 0$

Value: Any relevant value 1

22. Let P = probability of doublet

$P = \frac{1}{6}, q = \frac{5}{6}$ $\frac{1}{2}$

x:	0	1	2	3	$\frac{1}{2}$
P(x):	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$	2
xP(x):	0	$\frac{75}{216}$	$\frac{30}{216}$	$\frac{3}{216}$	$\frac{1}{2}$

Mean $= \Sigma xP(x) = \frac{108}{216} = \frac{1}{2}$ $\frac{1}{2}$

23. Let H_1 be the event that bolt is manufactured by machine A

H_2 be the event that bolt is manufactured by machine B

H_3 be the event that bolt is manufactured by machine C

and E be the event that bolt selected is defective

$P(H_1) = \frac{25}{100},\ P(H_2) = \frac{35}{100},\ P(H_3) = \frac{40}{100}$ 1

$P(E/H_1) = \frac{5}{100},\ P(E/H_2) = \frac{4}{100},\ P(E/H_3) = \frac{2}{100}$ 1

Reqd prob. is

$$P(H_2/E) = \frac{P(H_2)\cdot P(E/H_2)}{P(H_1)\cdot P(E/H_1) + P(H_2)P(E/H_2) + P(H_3)\cdot P(E/H_3)}$$ 1

$$= \frac{28}{69}$$ 1

SECTION D

24. AB = 8I 1

$$\Rightarrow \quad A^{-1} = \frac{1}{8}B = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$ $1\frac{1}{2}$

Given equation in matrix form is

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$ 1

$$\Rightarrow \quad AX = C$$

$$\Rightarrow \quad X = A^{-1}C$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$ 1

$$\Rightarrow \quad x = 3,\ y = -2,\ z = -1$$ $1\frac{1}{2}$

OR

$$A^2 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix},\ A^3 = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$ $1\frac{1}{2} + 1\frac{1}{2}$

$$A^3 - 6A^2 + 5A + 11I$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 11 \\ 0 & 0 & 11 \end{bmatrix}$$ $1\frac{1}{2}$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$ $1\frac{1}{2}$

25. f: $A \rightarrow A$

Let $x_1, x_2 \in A$ such that

$f(x_1) = f(x_2)$

$\Rightarrow \quad \dfrac{x_1 - 2}{x_1 - 1} = \dfrac{x_2 - 2}{x_2 - 1}$

$\Rightarrow \quad x_1 = x_2$ 2

$\Rightarrow$ f is one-one

Now $y = \dfrac{x-2}{x-1} \Rightarrow x - 2 = xy - y$

$\Rightarrow \quad x(y-1) = y - 2$

$\Rightarrow \quad x = \dfrac{y-2}{y-1}$ 1

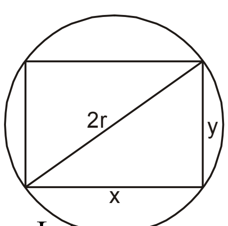
For each $y \in A = R - \{1\}$, there exists $x \in A$

Thus f is onto. Hence f is bijective $\frac{1}{2}$

and $f^{-1}(x) = \dfrac{x-2}{x-1}$ $\frac{1}{2}$

(i) $f^{-1}(x) = \dfrac{5}{6} \Rightarrow \dfrac{x-2}{x-1} = \dfrac{5}{6} \Rightarrow x = 7$ 1

(ii) $f^{-1}(2) = 0$ 1

26. 

Let x, y respectively be the sides of rectangle $\therefore\ y = \sqrt{4r^2 - x^2}$ ...(1)

$A = xy$ $\frac{1}{2}$

Let $Z = A^2 = 4x^2r^2 - x^4$ $1\frac{1}{2}$

$\dfrac{dZ}{dx} = 8r^2x - 4x^3$ 1

$\dfrac{dZ}{dx} = 0 \Rightarrow 4x(2r^2 - x^2) = 0$

$\Rightarrow x = \sqrt{2}r$ 1

$$\frac{d^2Z}{dr^2} = 8r^2 - 12x^2$$

$$\left.\frac{d^2Z}{dx^2}\right|_{x=\sqrt{2}r} = -16r^2 < 0$$ 1

$\Rightarrow$ Area is maximum when $x = \sqrt{2}r$

$\therefore$ $y = \sqrt{2}r$ (From (i))

i.e. $x = y$

Hence, Area is maximum when rectangle is a square 1

27. Equation of plane passing through $(-1, 3, 2)$

$a(x+1) + b(y-3) + c(z-2) = 0$ ...(i) 1

Required plane is perpendicular to $x + 2y + 3z = 5$

and $3x + 3y + z = 0$

$$\therefore \left.\begin{aligned} a + 2b + 3c &= 0 \\ 3a + 3b + c &= 0 \end{aligned}\right\}$$ 1+1

$$\Rightarrow \frac{a}{-7} = \frac{b}{8} = \frac{c}{-3}$$ 1

$\therefore$ Equation (i) $\Rightarrow$

$7x - 8y + 3z + 25 = 0$ 1

Vector Equation of plane is

$\vec{r} \cdot (7\hat{i} - 8\hat{j} + 3\hat{k}) = -25$ 1

OR

Equation of plane passing through $(1, 1, 4)$, $(3, -1, 2)$ and $(4, 1, -2)$ is

$$\begin{vmatrix} x-1 & y-1 & z-4 \\ 2 & -2 & -2 \\ 3 & 0 & -6 \end{vmatrix} = 0$$ 2

$\Rightarrow$ $2x + y + z = 7$ ...(i)

Equation of line passing through (3, –4, –5) and (2, –3, 1)

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = k$$ 1

$\Rightarrow$ $x = 3 - k, y = k - 4, z = 6k - 5$ ...(ii) 1

$(3 - k, k - 4, 6k - 5)$ lies on (i)

$6 - 2k + k - 4 + 6k - 5 - 7 = 0$

$\Rightarrow$ $k = 2$ 1

Eqn (ii) $\Rightarrow$ point of intersection is (1, –2, 7) 1

28. $\frac{dy}{dx} + y \cdot \cot x = 4x \operatorname{cosec} x$ ...(i)

Here P = cot x, I.F. = $e^{\int \cot x . dx} = e^{\log \sin x} = \sin x$ $1\frac{1}{2}$

Hence the solution is

$y \sin x = \int 4x \cdot dx$ 2

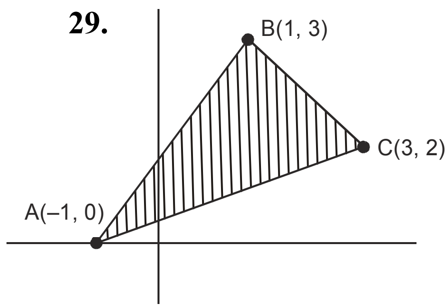
$\Rightarrow$ $y \sin x = 2x^2 + C$ $1\frac{1}{2}$

When $x = \frac{\pi}{2}, y = 0, C = -\frac{\pi^2}{2}$

$\therefore$ Requried solution is

$$y \sin x = 2x^2 - \frac{\pi^2}{2}$$ 1

29.



Equation of AB: $y = \frac{3}{2}(x+1)$

Equation of BC: $y = \frac{-1}{2}x + \frac{7}{2}$

Equation of AC: $y = \frac{1}{2}(x+1)$ $1\frac{1}{2}$

Required area

$$= \frac{3}{2}\int_{-1}^{1}(x+1)dx - \frac{1}{2}\int_{1}^{3}(x-7)\,dx - \frac{1}{2}\int_{-1}^{3}(x+1)dx \qquad 1\frac{1}{2}$$

$$= \frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1} - \frac{1}{2}\left[\frac{x^2}{2}-7x\right]_{1}^{3} - \frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{3} \qquad 1\frac{1}{2}$$

$= 3 + 5 - 4$

$= 4$ sq.units $1\frac{1}{2}$

OR

$$\int_{1}^{3}(3x^2 + e^{2x})dx$$

$$h = \frac{2}{n}, \text{ as } n \to \infty, h \to 0$$

$$\int_{1}^{3} f(x)\,dx = \lim_{\substack{h\to 0\\ n\to\infty}} h[f(x) + f(1+h) + f(1+2h) + \ldots + f(1+n)]$$

$$\int_{1}^{3}(3x^2 + e^{2x})dx = \lim_{\substack{h\to 0\\ n\to\infty}} 3h^3\Sigma(n-1)^2 + 6h^2.\Sigma(n-1) + 3nh + he^2[1 + e^{2h} + 2^{4h} + \ldots + f(1-n)] \qquad 2$$

$$= \lim_{\substack{h\to 0\\ n\to\infty}} \frac{24}{n^3}\cdot\frac{n(n-1)(2n-1)}{6} + \frac{24}{n^2}\cdot\frac{n(n-1)}{2} + 6 + he^2 1\cdot\frac{e^{2nh}-1}{e^{2h}-1} \qquad 2$$

$$= \lim_{\substack{h\to 0\\ n\to\infty}} \frac{24\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6} + \frac{24\left(1-\frac{1}{n}\right)}{2} + 6 + e^2 \qquad 1$$

$$= 26 + \frac{1}{2}(e^6 - e^2) \qquad 1$$

Alternately $f(x) = (3x^2 + e^{2x})$

$$f(x)dx = \lim_{h\to 0} \{[3(1)^2 + e^2] + [3(1+h)^2 + e^{2(1+h)}] + 3[3(1+2h)^2 + e^{2(1+2h)}] + \ldots +$$

$$[3(1+(n-1)h)^2] + e^{2(1+(n-1)h)}]\} \qquad 2$$

$$= \lim_{h\to 0} h\left[\frac{3n + 6hn(n-1)}{2} + \frac{3h^2 n(n-1)(2n-1)}{6} + e^2\{1 + e^{2h} + e^{4h} + \ldots + e^{2(n-1)h}\}\right] \qquad 2$$

$$= \lim_{h\to 0} h\left[3nh + 3nh\,(nh-h) + \frac{nh(nh-h)\,(2nh-h)}{2} + \frac{e^2 h\, e^{2nh} - 1}{e^{2h} - 1}\right] \qquad 1$$

$$= 6 + 12 + 8 + \frac{e^2(e^4 - 1)}{2} \qquad 1$$