BLUE PRINT/ UNITWISE WEIGHTAGE

FIRST PRE BOARD EXAM-MATHEMATICS XII (Series : PSLB) 2(1)

UNIT		CHAPTERS	1 MARK	2 MARKS	4 MARKS	6 MARKS	TOTAL MARKS
RELATIONS AND FUNCTIONS	1	RF				6(1)	10(2)
	2	IT			4(1)		
ALGEBRA	3	MAT		2(1)	4(1)		13(4)
	4	DET	1(1)			6(1)	
CALCULUS	5	CD		4(2)	4(1)		44(14)
	6	APPLN DER		2(1))	4(1)	6(1)	
	7	INT	1(1)	2(1)	8(2)		
	8	APP INT				6(1)	
	9	DE	1(1)	2(1)	4(1)		
VECTORS AND 3D GEOMETRY	10	VECT	1(1)	2(1)	4(1)		17(5)
	11	3D			4(1)	6(1)	
LPP	12	LPP		2(1)	4(1)		6(2)
PROBABILITY	13	PROB			4(1)	6(1)	10(2)
			4(4)	16(8)	44(11)	36(6)	100(29)

QUESTION PAPER DESIGN

S.No	Typology of Qns	VSA(1M)	SA-(2M)	LA-I (4M)	LA-II(6M)	MARKS	WEIGHTAGE
1	Remembering	2	2	2	1	20	20%
2	Understanding	1	3	4	2	35	35%
3	Application	1		3 (1 M-VBQ)	2	25	25%
4	HOTS		3	1		10	10%
5	Evaluation			1(VBQ)	1	10	10%
	Total	1(4)=4	2(8)=16	4(11)=44	6(6)=36	100	10%

QUESTIONWISE BREAK UP

TYPE OF QNS	Marks per Qn	Total No of Qns	Totala Marks
VSA	1	4	4
SA	2	8	16
LA-I	4	11	44
LA-II	6	6	36
TOTAL		29	100

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Series: PSLB

NAVODAYA VIDYALAYA SAMITI :: HYDERABAD REGION

FIRST PRE-BOARD EXAMIANATION 2017-18

CLASS XII **MATHEMATICS**

Time allowed : 3 hours **Maximum Marks: 100**

General Instructions:

i) All questions are compulsory.

ii) The question paper consists of 29 questions divided into four sections A,B,C and D. Section A comprises of 4 questions of 1 mark each, Section B comprises of 8 questions of 2 mark each. Section C comprises of 11 questions of 4 marks each and Section D comprises of 6 questions of 6 marks each.

iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

iv) There is no overall choice. However internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION A

1. Find the projection of the vector $\vec{a} = 2\hat{i} + 3j + 2k$ on the vector $b = 3\hat{i} - 2j + k$

2. Evaluate $\int e^{3\log x} x^6 dx$

3. Find the order and degree of the differential equation $\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$

4. If A is an invertible matrix of order 2 and det(A) = 4 then write the value of det(A^{-1})

SECTION B

5. If $ax + by^2 = \cos y$ then find $\frac{dy}{dx}$

6. Prove that $\left[\vec{a}, \vec{b}, \vec{c} + \vec{d}\right] = \left[\vec{a}, \vec{b}, \vec{c}\right] + \left[\vec{a}, \vec{b}, \vec{d}\right]$ for any four vectors $\vec{a}, \vec{b}, \vec{c}$ *and* $\vec{d}$

7. Anil wants to invest at least Rs. 12000 in bonds A and B. According to the rules, he has to invest at least Rs.2000 in bond A and at least Rs.4000 in bond B. If the rate of interest in bond A is 8% per annum and on bond B is 10% per annum,formulate this LPP to maximise the profit.

8. If A is a square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$

9. If $y = \sin^{-1} x$ show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

10. Form the differential equation of the family of circles touching the y axis at the origin

11. Evaluate $\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$

12. The sides of an equilateral triangle are increasing at the rate of 2cm/sec. Find the rate at which its area increases when the side is 10cm long.

SECTION C

13. A factory has 1,440 employees (men and women). For above-avarage results 18.75% of all men and 22.5% of all women received the bonus. If 20% of the total employees are awarded , find how many men and how many women are employed in the factory using matrices .What are the qualities required by an employee to produce above average results?

14. Find the intervals in which the function f given by $f(x) = \sin x + \cos x, \quad 0 \le x \le 2\pi$,is strictly increasing or strictly decreasing.

15. Evaluate Evaluate $\int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$

OR

Evaluate $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$

16. Let $\vec{a} = \hat{i} + 4j + 2k, \vec{b} = 3\hat{i} - 2j + 7k, and \quad \vec{c} = 2\hat{i} - j + 4k$ then find a vector $\vec{d}$ which is perpendicular to both $\vec{a} \quad and \quad \vec{b}$ and $\vec{c}.\vec{d} = 15$

17. Solve graphically : Maximise $Z = 6x + 8y \quad subject \quad to \quad 2x + y \le 30, \; x + 2y \le 24$, $x \ge 0, y \ge 0$

18. In answering a question on a multiple choice test, a student either knows the answer or guesses the answer. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that guesses. Assuming that a student who guesses the answer will be correct with probability $\frac{1}{4}$, what is the probability that the student knows the answer given that he answered correctly. What are values to be upheld while writing an examination?

19. Prove that $\tan^{-1}\left\{\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right\} = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x, \quad \frac{-1}{\sqrt{2}} \le x \le 1$

20. If the function $f(x) = \begin{cases} 3ax + b, if \quad x > 1 \\ 11, \text{if} \quad x = 1 \\ 5ax - 2b, \text{if} \quad x < 1 \end{cases}$ is continuous at x=1 then find the values of a and b

21. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}dx$

22. Find the general solution of the differential equation $ydx-(x+2y^2)dy=0$

OR

Find the particular solution of the differential equation

$ye^y dx = \left(y^3+2xe^y\right)dy, \quad y(0)=1$

23. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3)

OR

If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $x-3=\frac{y-k}{2}=z$ intersect, find the value of k. and hence find the equation of the plane containing these lines

SECTION D

24. If $a+b+c\neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=0$ then using properties ,prove that $a=b=c$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix}=(1+pxyz)(x-y)(y-z)(z-x)$$

25. Find the area of the region enclosed between the two circles $x^2+y^2=4$ and $(x-2)^2+y^2=4$

26. From the point P(1,2,4) , a perpendicular is drawn on the plane 2x + y + 2z + 15 = 0. Find the equation, the length and the coordinates of the foot of the perpendicular.

27. A binary operation $*$ defined on the set $\mathbb{R}$ the set of real numbers by

$a*b=\begin{cases} a, if \quad b=0 \\ |a|+b, b\neq 0 \end{cases}$ .If at least one of a and b is 0 then prove that $a*b=b*a$ .Check whether $*$ is commutative. Find the identity element for $*$ if it exists.

OR

Show that the relation S in the set A = $\{x \in Z : \; 0 \;\leq\; X \;\leq 12\}$ given by

S = $\{(a,b): \; a,b \;\in Z, |a-b| is\ divisible\ by\ 4\ \}$ is an equivalence relation. Also find the set of all elements related to 1.

28. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base

OR

Find the coordinates of a point on the parabola $y = x^2 + 7x + 2$ which is closest to the straight line y=3x-3

29. Two cards are drawn simultaneously from a well-shuffled pack of 52 cards. Find the mean and standard deviation of the number of kings.

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Series: PSLB

Marking Scheme — Mathematics (XII)

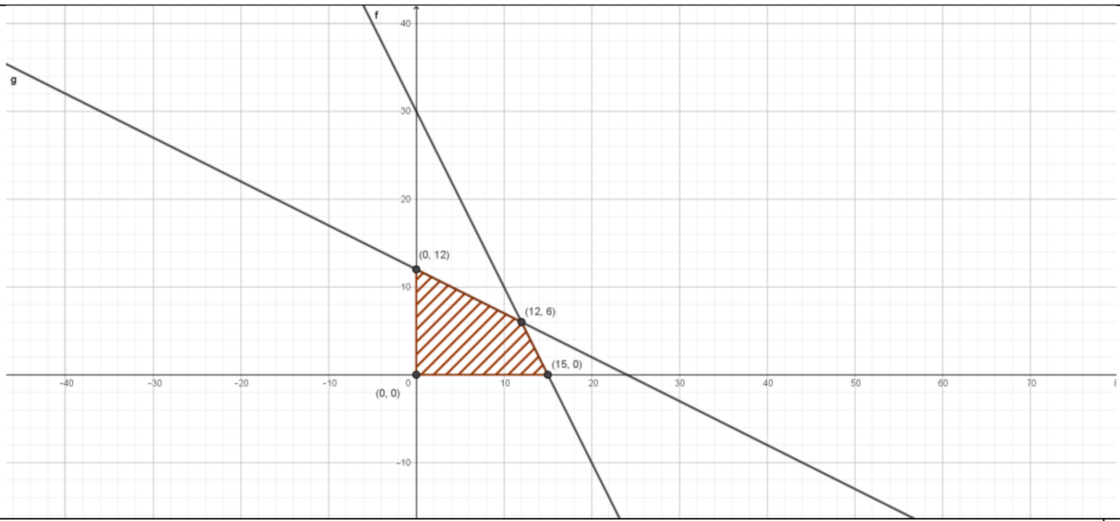
1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.
2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.
3. Alternative methods are accepted. Proportional marks are to be awarded.
4. In question (s) on differential equations, constant of integration has to be written
5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out
6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it

EXPECTED ANSWER/VALUE POINTS

	SECTION A	
1	Formula Projection $\frac{\vec{a}.\vec{b}}{\lvert\vec{b}\rvert}$ Getting the answer $\frac{2}{\sqrt{14}}$	½ 1/2
2	$\int x^3 x^6 dx$	1/2
	Getting final ans $\frac{x^{10}}{10}+c$	1/2
3	Order 1	1/2
	Degree not defined	1/2
4	Det of $A^{-1}=\frac{1}{\lvert A\rvert}$ Getting ans $\frac{1}{4}$	½ **1/2**
	SECTION B	
5	a+2by $\frac{dy}{dx}$ $=-\sin$ y $\frac{dy}{dx}$ $-$	**1**
	$\frac{dy}{dx}=\frac{-a}{\sin y+2by}$	**1**
6.	$\left[\vec{a},\vec{b},\vec{c}+\vec{d}\right]=\left(\vec{a}\times\vec{b}\right).\left(\vec{c}+\vec{d}\right)$	**1/2**
	$=$ **=** $\left(\vec{a}\times\vec{b}\right).\vec{c}+\left(\vec{a}\times\vec{b}\right).\vec{d}$	**1**
	= $\left[\vec{a},\vec{b},\vec{c}\right]+\left[\vec{a},\vec{b},\vec{d}\right]$	½

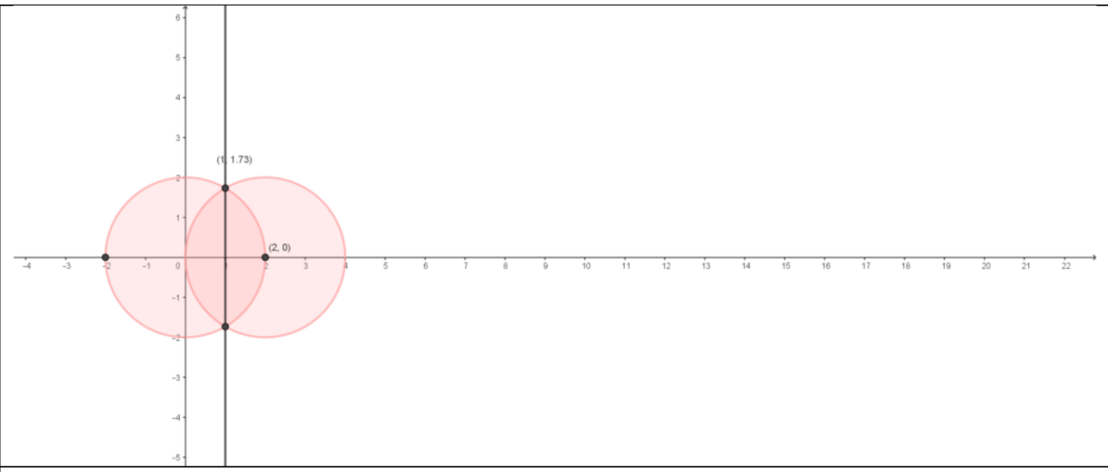
7.	**Amount to bond A =x** **Amount in bond B=y** **Maximixe** $Z=\frac{8x}{100}+\frac{10y}{100}$	**1**
	$x+y\ge 12000$ $x\ge 2000$ $y\ge 4000$ $x\ge 0; y\ge 0$	**1**
8.	$A^2=A\Rightarrow A=I$	**1/2**
	$\left(I+A\right)^3=8I$	**1**
	RHS=8I	**1/2**
9	$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$	**1**
	Differentiating to reach $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0$	**1**
10	**Writing the equation of the family of circles** $x^2+y^2=2ax$	**1**
	To get the differential equation $y^2-x^2-2xyy_1=0$	**1**
11	**Substitute** $e^x x=u$	**1/2**
	$e^x(1+x)dx=du$	**1/2**
	$I=\int\sec^2 u du$	**1/2**
	$=\tan u+c=\tan(xe^x)+c$ **)**	**1/2**
12	$A=\frac{\sqrt{3}}{4}a^2$	**1/2**
	$\frac{dA}{dt}=\frac{\sqrt{3}}{4}2a.\frac{da}{dt}$	**1**
	To get ans $10\sqrt{3}\quad cm^2/s$	
	Section C	
13	**x men and y women** $\frac{18.75}{100}x+\frac{22.5}{100}y=\frac{20}{100}\left(x+y\right)$	**1/2**
	x-2y=0 **x+y=1440**	**1**
	$\begin{bmatrix}1 & -2\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 1440\end{bmatrix}$	**1/2**
	Findind det A=3,and $A^{-1}=\frac{1}{3}\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}$	**½+1/2**
	X=960 any y=480	**1/2**
	values	**1/2**

14	$f'(x)=\cos x-\sin x$	1/2
	For finding intervals $\left[0,\frac{\pi}{4}\right),\left(\frac{\pi}{4},\frac{5\pi}{4}\right),\left(\frac{5\pi}{4},2\pi\right]$	1
	Increasing in $\left[0,\frac{\pi}{4}\right)$, and $\left(\frac{5\pi}{4},2\pi\right]$	1 1/2
	Decreasing in $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$	1
15	**Put sinx =t then cosdx=dt**	1/2
	$I=\int\frac{dt}{(1-t)(2-t)}$	1/2
	Resolving into partial fraction $\frac{A}{1-t}+\frac{B}{2-t}$ $A=1,B=-1$	1
	Integrating and getting the answer $\log\left\|\frac{2-\sin x}{1-\sin x}\right\|+c$	2
	Or	
	$I=\int\frac{(\sin^2 x+\cos^2 x)^3-3\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x)}{\sin^2 x\cos^2 x}dx$	1
	= $\int\left(\frac{1}{\sin^2 x\cos^2 x}-3\right)dx$	1
	$\int\left(\frac{\sin^2 x+\cos^2 x}{\sin^2 x\cos^2 x}-3\right)dx$	1
	=tanx-cotx-3x+c	1
16	**Finding** $\vec{a}\times\vec{b}=32\hat{i}-j-14k$	1 1/2
	Writing $\vec{d}\quad as\quad \vec{d}=\lambda\left(\vec{a}\times\vec{b}\right)$	1/2
	Solving $\vec{c}.\vec{d}=15\Rightarrow\lambda=\frac{5}{3}$	1 1/2
	Writing the ans $\vec{a}=\frac{5\left(32\hat{i}-j-14k\right)}{3}$	1/2

17		
	For drawing graph	**1 1/2**
	For corner points (0,0), (15,0), (12,6), (0,12)	**1**
	To find the value of the objective function at corner points **As 0, 90, 120 and 96 and finding answer as 120**	
18	**Let K knows the answer G Guesses the answer, C correct answer** **Required to find P(K/C)**	**1/2**
	$P(K/C)=\dfrac{P(C/K)P(K)}{P(C/K)P(K)+P(C/G)P(G)}$	**1**
	$P(K)=\dfrac{3}{4}$ $P(G)=\dfrac{1}{4}$ $P\left(C/K\right)=1$ $P\left(C/G\right)=\dfrac{1}{4}$	**1**
	Substituting and simplifying to get correct ans = 12/13	**1 1/2**
19	$x=\cos 2\theta$	**1/2**
	Reducing into $\tan^{-1}\left(\dfrac{1-\tan\theta}{1+\tan\theta}\right)$	**2 1/2**
	Simplifying to get $\dfrac{\pi}{4}-\dfrac{1}{2}\cos^{-1}x$	**1**
20	**LHL=RHL=f(1)** **3a+b=11** **5a-2b=11**	**2**
	Solving a=2 **b=5**	**2**
21	$\int_a^b f(x)dx=\int_a^b f(\mathrm{a}+\mathrm{b}-x)dx$	**1**

	$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	**1**
	$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}{\sqrt{\sin x\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}+\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}dx$	**1**
	2I= $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}dx$ **=** $\frac{\pi}{6}\Rightarrow I=\frac{\pi}{12}$	**1**
22	**Writing as** $\frac{dx}{dy}-\frac{x}{y}=2y$ **which Linear DE**	**1**
	IF= $\frac{1}{y}$	**1**
	Solution is x(IF)= $\int Q(y)(IF)dy+c$	**1/2**
	Simplify $\frac{x}{y}=\int 2ydy+c$ **to get** $\Rightarrow\ x=2y^2+cy$	**1 1/2**
	OR	
	Writing as $\frac{dx}{dy}-\frac{2x}{y}=\frac{y^2}{e^y}$ **which is linear DE**	**1**
	IF= $\frac{1}{y^2}$	**1**
	Solution x. $\frac{1}{y^2}=\int e^{-y}dy+c$	**1/2**
	X= $-y^2e^{-y}$ **+** cy^2	**1/2**
	C= $\frac{1}{e}$	**1/2**
	Solution is $x=-y^2e^{-y}+\frac{y^2}{e}$	**1/2**
23	**Writing the general point as (3k-1,2k-1,2k+3)**	**1**
	Using distance formula and writing $(3k-3)^2+(2k-4)^2+(2k)^2=25$	**1**
	Solving and getting k=0,2	**1**
	Points on the line are (-2,-1,3) and (4,3,7)	**1**
	OR	
	Simplifying $\begin{vmatrix}1-3 & -1-k & 1-0\\ 2 & 3 & 4\\ 1 & 2 & 1\end{vmatrix}=0$ **getting** $k=\frac{9}{2}$	**2**

	Simplifying $\begin{vmatrix} x-3 & y-\frac{9}{2} & z-0 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$ **to get 5x-2y-z=6**	**2**
	SECTION D	
24	$c_1 \to c_1 + c_2 + c_3$	**1**
	$(a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$	**1**
	$R_2 \to R_2 - R_1$ **and** $R_3 \to R_3 - R_1$	**1**
	$(a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0$	**1**
	Simplifying to reach $-\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]=0$	**1**
	a=b=c	**1**
	OR	
	$R_2 \to R_2 - R_1 and R_3 \to R_3 - R_1$	**1**
	$\begin{vmatrix} x & x^2 & 1+px^2 \\ y-x & y^2-x^2 & p\left(y^3-x^3\right) \\ z-x & z^2-x^2 & p\left(z^3-x^3\right) \end{vmatrix}$	**1**
	$(y-x)(z-x)\begin{vmatrix} x & x^2 & 1+px^2 \\ 1 & y+x & p\left(y^2+xy+x^2\right) \\ 1 & z^2-x^2 & p\left(z^2+xz+x^2\right) \end{vmatrix}$	**1**
	Applying $R_3 \to R_3 - R_2$	**1/2**
	$(y-x)(z-x)(z-y)\begin{vmatrix} x & x^2 & 1+px^2 \\ 1 & y+x & p\left(y^2+xy+x^2\right) \\ 0 & 1 & p(x+y+z) \end{vmatrix}$	**1 1/2**
	Expanding to reach $(x-y)(y-z)(z-x)(1+pxyz)$	**1**

25		
	For correct diagram	1 1/2
	To find point of intersection $\left(1,\sqrt{3}\right)$	1
	$2\left[\int_0^1 \sqrt{4-\left(x-2\right)^2}\,dx+\int_1^2 \sqrt{4-x^2}\,dx\right]$	1 1/2
	Simplifying to get $\frac{8\pi}{3}-2\sqrt{3}$ sq units	2
26	Equation of the perpendicular to the plane is $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-4}{2}=\lambda$	1
	General point on this perpendicular is $\left(2\lambda+1,\lambda+2,2\lambda+4\right)$	1
	Since the point lies on plane $2\left(2\lambda+1\right)+\left(\lambda+2\right)+2\left(2\lambda+4\right)+15=0$	1 1/2
	Solving $\lambda=-3$	1
	Foot of perpendicular (-5,-1.-2)	1/2
	Distance =9 units	1
27	For $a=0,b\neq 0$ then to get $a*b=b*a$	1
	For $a\neq 0,b=0$ then to get $a*b=b*a$	1
	For $a=0,b=0$ then to get $a*b=b*a$	1
	Now $a\neq 0,b\neq 0$ then $a*b$ $may\,not\ \ be\ \ equal\ \ to\ \ b*a$ Numerical ex: a=-1,b=2	1
	Identity element 0	2
	OR	
	For showing reflexivity (4 divides $\lvert a-a\rvert$)	1
	Symmetric 4 divides $\lvert a-b\rvert$ then 4 divides $\lvert b-a\rvert$ as $\lvert a-b\rvert$ = $\lvert b-a\rvert$	1 1/2
	Transitive 4 divides $\lvert a-b\rvert$ and $\lvert b-c\rvert$ then $a-b=\pm 4m, b-c=\pm 4n$ $\Rightarrow a-b+b-c=\pm 4(m+n)=\pm 4k \ \ \Rightarrow 4divides\lvert a-c\rvert$	2
	Elements related to 1 are 1,5,9	1 1/2
28	Volume of cone $\frac{1}{3}\pi r^r h$ CSA= $=A=\pi rl$	1

	Z= $A^2 = \pi^2 r^4 + \frac{9V^2}{r^2}$	1
	$\frac{dZ}{dr} = 0 \Rightarrow r = h/\sqrt{2}$	2
	$\frac{d^2Z}{dr^2} = 12\pi^2 r^2 + \frac{54V^2}{r^4}$ **which is positive when** $r = \frac{h}{\sqrt{2}}$	1
	$\Rightarrow hence\ \ CSA\ \ will\ \ be\ \ \min imum\ \ if\ \ h = \sqrt{2}r$	1
	OR	
	Y= $\left(x + \frac{7}{2}\right)^2 - \frac{41}{4}$	1
	Any point on it is $\left(t - \frac{7}{2}, t^2 - \frac{41}{4}\right)$	11/2
	Eqn to line y-3x+3	
	Distance= $z = \frac{\left(t^2 - \frac{41}{4}\right) - 3\left(t - \frac{7}{2}\right) + 3}{\sqrt{10}}$	1
	$\frac{dz}{dt} = \frac{1}{\sqrt{10}}(2t - 3)$	1
	$\frac{dz}{dt} = 0 \Rightarrow t = \frac{3}{2}$	1/2
	$\frac{d^2z}{dt^2} = \frac{2}{\sqrt{10}} > 0$	1/2
	Minimum distance= $\frac{1}{\sqrt{10}}$	1/2
29	**X =number of cards drawn be king Then possible values of X are 0,1,2**	
	X | **0** | **1** | **2** **P(X)** | $\frac{188}{221}$ | $\frac{32}{221}$ | $\frac{1}{221}$	2
	Mean= $\mu = \sum x_i p_i$ **=** $\frac{2}{13}$ $E(X^2) = \sum x_i^2 pi = \frac{36}{221}$	1 1
	SD= $\sqrt{Variance}$ **=** $\sqrt{\left(E(X^2\right) - (E(X))^2}$ **=** $\frac{\sqrt{6800}}{221} = 0.37$	1
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NAVODAYA VIDYALAYA SAMITI :: HYDERABAD REGION

FIRST PRE-BOARD EXAMIANATION 2017-18

CLASS XII **MATHEMATICS**

Time allowed : 3 hours **Maximum Marks: 100**

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iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

iv) There is no overall choice. However internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION A

1. If $\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}.\vec{b}\right)^2=225$ and $\left|\vec{a}\right|=5$ then find the value of $\left|\vec{b}\right|$

2. Evaluate $\int \text{logx}\, dx$

3. Find the order and degree of the differential equation $\sqrt{\frac{dy}{dx}}+\left(\frac{d^2y}{dx^2}\right)=0$

4. If A is square matrix of order 3 and |A|=5, then what is the value of |A Adj.A|

SECTION B

5. If $ax^2+2hxy+by^2=0$ then find $\frac{dy}{dx}$

6. Prove that $\left[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}\right]=2\left[\vec{a},\vec{b},\vec{c}\right]$ for any three vectors $\vec{a},\vec{b},\vec{c}$

7. Solve the LPP graphically : Maximise Z=3x+4y subject to $x+y\le 4, x\ge 0, y\ge 0$.

8. If A=$\begin{pmatrix} 2a & -1 \\ -8 & 3 \end{pmatrix}$ is a singular matrix then find the value of a

9. If $y=\left(\tan^{-1}x\right)^2$ show that $(1+x^2)^2\frac{d^2y}{dx^2}+2x\left(1+x^2\right)\frac{dy}{dx}=2$

10. Form the differential equation of the family of lines passing through the origin .

11. Evaluate $\int e^x(\sin x+\cos x)dx$

12. The sides of a cube is increasing at the rate of 2cm/sec. Find the rate at which its Volume and Surface area increase when the side is 4cm long.

SECTION C

13. If A=$\begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$

14. Find the intervals in which the function f given by $f(x) = 2x^3 - 15x^2 + 36x + 42$,is increasing or decreasing.

15. Evaluate $\int \frac{2x}{(1+x)(2+x^2)} dx$

OR

Evaluate $\int \frac{1-e^{-2x}}{1+e^{-2x}} dx$

16. Let $\vec{a} = \hat{i} + 2j + k, \vec{b} = 3\hat{i} + 2j + 5k$ then express $\overrightarrow{b}$ in the form $\vec{b} = \vec{c} + \vec{d}$ where $\vec{c}$ *is parallel to* $\vec{a}$ and $\vec{d}$ is perpendicular to $\vec{a}$.

17. A dietician wishes to mix two types of foods in such a way that Vitamin contents of the mixture contains at least 8 units of Vitamin A and 10 units of Vitamin C. Food I contain 2 unit per kg of Vitamin A and 1 unit per kg of Vitamin C. Food II contains 1 unit per kg of Vitamin A and 2 unit per kg of Vitamin C. It costs rs.50 per kg to purchase food I and rs.70 per kg to purchase food II. Formulate this problem as a Linear Programming problem to minimise the cost of such a mixture.

18. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. We have to speak truth always. Why?

19. If $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ then find the value of x

20. If the function $f(x) = \begin{cases} \frac{k \cos x}{\pi = 2x}, at \quad , x = \frac{\pi}{2} \\ 3, if \quad x = \frac{\pi}{2} \end{cases}$ is continuous then find the value of k

21. Prove that Evaluate $\int_a^b f(x)dx = \int_a^b f(\text{a}+\text{b}-x)dx$

22. Show that the differential equation (x-y) dy = (x+y) dx is homogenous and solve it.

OR

Solve the differential equation $\frac{dy}{dx}$ + y cotx = 2x + x^2 cotx (x≠0). It is given that y=0, when x= $\frac{\pi}{2}$.

23. Find the shortest distance between the following pair of skew lines

$\frac{x-1}{2} = \frac{2-y}{3} = \frac{z+1}{4}$ and $\frac{x+2}{-1} = \frac{y-3}{2} = \frac{z}{3}$

OR

Show that $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $x-3=\frac{2y-9}{4}=z$ intersect, and hence find the equation of the plane containing these lines.

SECTION D

24. Prove that $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = (a^3+b^3)^2$

OR

Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

25. Find the area of the region enclosed between the two curves $x^2=4y$ and $y^2=4x$

26. Find the distance of the point, represented by $3\bar{i} - 2\bar{j} + \bar{k}$ from the plane 3x+y–z+2=0 measured parallel to the line $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z-1}{1}$. Also, find the foot of the perpendicular from the given point upon the given plane.

27. Let f: N → N be defined by $f(n)=\begin{cases} \frac{(n+1)}{2}, & if\ n\ is\ odd \\ \frac{n}{2}, & if\ n\ is\ even \end{cases}$ state whether the function f is bijective . Justify your answer.

OR

Let f: W → W be defined as $f(n)=\begin{cases} n-1, & if\ n\ is\ odd \\ n+1, & if\ n\ is\ even \end{cases}$ Then show that f is invertible. Also find the inverse of f.

28. Show that the height of a cylinder of maximum volume that can be inscribed in a sphere of radius 'R' is $\frac{2R}{\sqrt{3}}$

OR

A wire of length 36 cm is cut into two pieces. One piece is to be made into a square and another into an equilateral triangle. Find the length of each pieces so that the combined area is to be minimum

29. Find the Mean, Variance and the Standard Deviation of the number of doublets in three throws of a pair of dice