SAMPLE QUESTION PAPER

MATHEMATICS (041)

CLASS XII – 2017-18

Time allowed: 3 hours ***Maximum Marks: 100***

General Instructions:

(i) ***All*** *questions are compulsory.*
(ii) *This question paper contains* ***29*** *questions.*
(iii) *Question* ***1- 4*** *in* ***Section A*** *are very short-answer type questions carrying* ***1*** *mark each.*
(iv) *Questions* ***5-12*** *in* ***Section B*** *are short-answertype questions carrying* ***2*** *marks each.*
(v) *Questions* ***13-23*** *in* ***Section C*** *are long-answer-****I*** *type questions carrying* ***4*** *marks each.*
(vi) *Questions* ***24-29*** *in* ***Section D*** *are long-answer-****II*** *type questions carrying* ***6*** *marks each.*

	Section A **Questions 1 to 4 carry 1 mark each.**
1.	Let $A=\{1,2,3,4\}$. Let R be the equivalence relation on $A\times A$ defined by $(a,b)R(c,d)$ iff $a+d=b+c$. Find the equivalence class $[(1,3)]$.
2.	If $A=\left[a_{ij}\right]$ is a matrix of order 2×2, such that $\lvert A\rvert=-15$ and C_{ij} represents the cofactor of a_{ij}, then find $a_{21}c_{21}+a_{22}c_{22}$
3.	Give an example of vectors $\vec{a}$ and $\vec{b}$ such that $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert$ but $\vec{a}\neq\vec{b}$.
4.	Determine whether the binary operation $*$ on the set **N** of natural numbers defined by $a*b=2^{ab}$ is associative or not.
	Section B **Questions 5 to 12 carry 2 marks each**
5.	If $4\sin^{-1}x+\cos^{-1}x=\pi$, then find the value of x.
6.	Find the inverse of the matrix $\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}$. Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$.

7.	Prove that if $\frac{1}{2} \le x \le 1$ then $\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right] = \frac{\pi}{3}$
8.	Find the approximate change in the value of $\frac{1}{x^2}$, when x changes from $x = 2$ to $x = 2.002$
9.	Find $\int e^x \frac{\sqrt{1+\sin 2x}}{1+\cos 2x} dx$
10.	Verify that $ax^2 + by^2 = 1$ is a solution of the differential equation $x(yy_2 + {y_1}^2) = yy_1$
11.	Find the Projection (vector) of $2\hat{i} - \hat{j} + \hat{k}$ on $\hat{i} - 2\hat{j} + \hat{k}$.
12.	If A and B are two events such that $P(A) = 0.4,\ P(B) = 0.8$ and $P(B\|A) = 0.6$, then find $P(A\|B)$.
	Section C **Questions 13 to 23 carry 4 marks each.**
13.	$If\ \ \Delta = \begin{vmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & 1 & a \end{vmatrix} = -4$ $then\ find\ the\ value\ of\ \begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}.$
14.	Find 'a' and 'b', if the function given by $f(x) = \begin{cases} ax^2 + b, \text{ if } x < 1 \\ 2x+1, \text{ if } x \ge 1 \end{cases}$ is differentiable at $x = 1$ **OR** Determine the values of 'a' and 'b' such that the following function is continuous at $x = 0$: $f(x) = \begin{cases} \frac{x + \sin x}{\sin(a+1)x}, \text{ if } -\pi < x < 0 \\ 2, \text{if } x = 0 \\ 2\frac{e^{\sin bx} - 1}{bx}, \text{if } x > 0 \end{cases}$

15.	If $y = \log(\sqrt{x} + \frac{1}{\sqrt{x}})^2$, then prove that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.
16.	Find the equation(s) of the tangent(s) to the curve $y = (x^3 - 1)(x - 2)$ at the points where the curve intersects the x –axis. **OR** Find the intervals in which the function $f(x) = -3\log(1+x) + 4\log(2+x) - \frac{4}{2+x}$ is strictly increasing or strictly decreasing.
17.	A person wants to plant some trees in his community park. The local nursery has to perform this task. It charges the cost of planting trees by the following formula: $C(x) = x^3 - 45x^2 + 600x,$ Where x is the number of trees and C(x) is the cost of planting x trees in rupees. The local authority has imposed a restriction that it can plant 10 to 20 trees in one community park for a fair distribution. For how many trees should the person place the order so that he has to spend the least amount? How much is the least amount? **Use calculus to answer these questions. Which value is being exhibited by the person?**
18.	Find $\int \frac{\sec x}{1 + \cos ecx} dx$
19.	Find the particular solution of the differential equation : $ye^y dx = (y^3 + 2xe^y)dy, y(0) = 1$ **OR** Show that $(x - y)dy = (x + 2y)dx$ is a homogenous differential equation. Also, find the general solution of the given differential equation.
20.	If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}, and\ hence\ show\ that\ \left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = 0.$
21.	Find the equation of the line which intersects the lines $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1).

22.	Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they came from Bag III.
23.	Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean and variance of the distribution.
	Section D **Questions 24 to 29 carry 6 marks each.**
24.	If the function $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=2x-3$ and $g:\mathbb{R}\to\mathbb{R}$ by $g(x)=x^3+5$, then find $f\circ g$ and show that $f\circ g$ is invertible. Also, find $(f\circ g)^{-1}$, hence find $(f\circ g)^{-1}(9)$. **OR** A binary operation $*$ is defined on the set $\mathbb{R}$ of real numbers by $a*b=\begin{cases} a, \text{ if } b=0 \\ \lvert a\rvert+b, \text{ if } b\neq 0\end{cases}$. If at least one of a and b is 0, then prove that $a*b=b*a$. Check whether $*$ is commutative. Find the identity element for $*$, if it exists.
25.	If $A=\begin{bmatrix}3 & 2 & 1\\ 4 & -1 & 2\\ 7 & 3 & -3\end{bmatrix}$, then find A^{-1} and hence solve the following system of equations: $3x+4y+7z=14, 2x-y+3z=4, x+2y-3z=0$ **OR** If $A=\begin{bmatrix}2 & 1 & 1\\ 1 & 0 & 1\\ 0 & 2 & -1\end{bmatrix}$, find the inverse of A using elementary row transformations and hence solve the following matrix equation $XA=\begin{bmatrix}1 & 0 & 1\end{bmatrix}$.
26.	Using integration, find the area in the first quadrant bounded by the curve $y=x\lvert x\rvert$, the circle $x^2+y^2=2$ and the y-axis

27.	Evaluate the following: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$ **OR** Evaluate $\int_{-2}^{2} (3x^2-2x+4)dx$ as the limit of a sum.
28.	*Find the distance of point* $-2\hat{i}+3\hat{j}-4\hat{k}$ from the line $\vec{r}=\hat{i}+2\hat{j}-\hat{k}+\lambda(\hat{i}+3\hat{j}-9\hat{k})$ measured parallel to the plane: $x - y + 2z - 3 = 0$.
29.	A company produces two different products. One of them needs 1/4 of an hour of assembly work per unit, 1/8 of an hour in quality control work and Rs1.2 in raw materials. The other product requires 1/3 of an hour of assembly work per unit, 1/3 of an hour in quality control work and Rs 0.9 in raw materials. Given the current availability of staff in the company, each day there is at most a total of 90 hours available for assembly and 80 hours for quality control. The first product described has a market value (sale price) of Rs 9 per unit and the second product described has a market value (sale price) of Rs 8 per unit. In addition, the maximum amount of daily sales for the first product is estimated to be 200 units, without there being a maximum limit of daily sales for the second product. Formulate and solve graphically the LPP and find the maximum profit.

Marking Scheme (Mathematics XII 2017-18)

Sr. No.	Answer	Mark(s)
	Section A	
1.	$\left[(1,3)\right]=\{(x,y)\in A\times A: x+3=y+1\}=\{(x,y)\in A\times A: y-x=2\}=\{(1,3),(2,4)\}$	[1]
2.	-15	[1]
3.	$\vec{a}=\hat{i}\ ,\ \vec{b}=\hat{j}\cdot$ (or any other correct answer)	[1]
4.	$(1*2)*3=2^2*3=2^{12},1*(2*3)=1*2^6=2^{64}\ \therefore (1*2)*3\neq 1*(2*3).$ Hence, $*$ is not associative.	[1]
	Section B	
5.	$\Rightarrow 4\sin^{-1}x+\frac{\pi}{2}-\sin^{-1}x=\pi$ $\Rightarrow 3\sin^{-1}x=\frac{\pi}{2}\Rightarrow \sin^{-1}x=\frac{\pi}{6}\Rightarrow x=\sin\frac{\pi}{6}=\frac{1}{2}$	[1] [1]
6.	$\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}^{-1}=\frac{1}{9-10}\begin{bmatrix}-3 & -2\\ -5 & -3\end{bmatrix}=\begin{bmatrix}3 & 2\\ 5 & 3\end{bmatrix}$ $P=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}\begin{bmatrix}3 & 2\\ 5 & 3\end{bmatrix}=\begin{bmatrix}13 & 8\\ 1 & 1\end{bmatrix}$	[1 + ½] [½]
7.	Let $\cos^{-1}x=\theta.$ Then $\forall x\in\left[\frac{1}{2},1\right],\theta\in\left[0,\frac{\pi}{3}\right],x=\cos\theta$ The given expression on LHS $=\theta+\cos^{-1}\left[\frac{\cos\theta}{2}+\frac{\sqrt{3}\sin\theta}{2}\right]=\theta+\cos^{-1}\left[\cos(\theta-\frac{\pi}{3})\right]=\theta+\cos^{-1}\left[\cos(\frac{\pi}{3}-\theta)\right]$ $=\theta+\frac{\pi}{3}-\theta \quad \left(\because 0\le\frac{\pi}{3}-\theta\le\frac{\pi}{3}\right)$ $=\frac{\pi}{3}=RHS$	[½] [1] [½]
8.	Let $y=\frac{1}{x^2}$. Then $\frac{dy}{dx}=\frac{-2}{x^3}.$ $dy=(\frac{dy}{dx})_{x=2}\times\Delta x=\frac{-2}{2^3}\times 0.002=-0.0005.$ y decreases by 0.0005.	[1/2] [1] [1/2]
9.	$\int e^x\frac{\sqrt{1+\sin 2x}}{1+\cos 2x}dx=\int e^x\frac{\sqrt{(\sin x+\cos x)^2}}{2\cos^2 x}dx$ $=\frac{1}{2}\int e^x(\frac{\sin x}{\cos^2 x}+\frac{\cos x}{\cos^2 x})dx=\frac{1}{2}\int e^x(\sec x+\sec x\tan x)dx$ $=\frac{1}{2}e^x\sec x+c\ \left[\because \int e^x(f(x)+f'(x))dx=e^x f(x)+c\right]$	[1/2] [1/2] [1]

10.	$ax^2+by^2=1 \Rightarrow 2ax+2byy_1=0 \Rightarrow ax+byy_1=0$ (1) $\Rightarrow a+b[yy_2+{y_1}^2]=0 \Rightarrow a=-b[yy_2+{y_1}^2]$ (2) Substituting this value, for a in the equation (1), we get, $-b[yy_2+{y_1}^2]x+byy_1=0 \Rightarrow x[yy_2+{y_1}^2]=yy_1$. Hence verified	[1/2] [1] [1/2]
11.	$\vec{a}=2\hat{i}-\hat{j}+\hat{k},\vec{b}=\hat{i}-2\hat{j}+\hat{k},\vec{a}\cdot\vec{b}=5,\left\|\vec{b}\right\|=\sqrt{6}.$ The required Projection (vector) of $\vec{a}$ on $\vec{b}=\dfrac{\vec{a}\cdot\vec{b}}{\left\|\vec{b}\right\|^2}\vec{b}$ $=\dfrac{5}{6}(\hat{i}-2\hat{j}+\hat{k}).$	[1/2] [1] [1/2]
12.	$P(B/A)=\dfrac{P(A\cap B)}{P(A)} \Rightarrow P(A\cap B)=0.4\times 0.6=0.24$ $P(A/B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{0.24}{0.80}=\dfrac{3}{10}$	[1] [1]
	Section C	
13.	$Let\ \Delta_1=\begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}=\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$ Where C_{ij} = the cofactor of a_{ij} and a_{ij} = the (i, j)th element of determinant Δ. $We\ know\ that\ \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}=\Delta^2$ $\therefore\ \Delta_1=\Delta^2=(-4)^2=16$	[2] [1] [1]
14.	Since, f is differentiable at 1, f is continuous at 1. Hence, $\lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^+}(2x+1)=3$ $\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^-}(ax^2+b)=a+b$ f(1) = 3 As f is continuous at 1, we have a + b =3 .... (1) $Lf'(1)=\lim\limits_{h\to 0^+}\dfrac{f(1-h)-f(1)}{-h}=\lim\limits_{h\to 0^+}\dfrac{a(1-h)^2+b-3}{-h}$ $=\lim\limits_{h\to 0^+}\dfrac{a+ah^2-2ah+b-3}{-h}=\lim\limits_{h\to 0^+}(-ah+2a)$ (using (1)) = 2a $Rf'(1)=\lim\limits_{h\to 0^+}\dfrac{f(1+h)-f(1)}{h}=\lim\limits_{h\to 0^+}\dfrac{2(1+h)+1-3}{h}=2.$ As f is differentiable at 1, we have 2 a = 2, i. e., a = 1 and b = 2. **OR**	[1] [1/2] [1/2] [1/2] [1/2] [1/2] [1/2]

	$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x+\sin x}{\sin(a+1)x}$	[1/2]
	$= \lim_{x\to 0^-} \frac{1+\frac{\sin x}{x}}{\frac{\sin(a+1)x}{(a+1)x}(a+1)} = \frac{2}{a+1}$	[1/2]
	$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2\frac{e^{\sin bx}-1}{bx}$	[1/2]
	$= \lim_{x\to 0^+} 2\frac{e^{\sin bx}-1}{\sin bx}\times\frac{\sin bx}{bx} = 2$	[1/2]
	f(0) = 2.	[1/2]
	For the function to be continuous at 0, we must have $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0)$	[1/2]
	i.e., we must have $\frac{2}{a+1} = 2 \Rightarrow a = 0;$ b may be any real number other than 0.	[1]
15.	$y = \log(\sqrt{x}+\frac{1}{\sqrt{x}})^2 = 2\log(\frac{x+1}{\sqrt{x}}) = 2[\log(x+1) - \frac{1}{2}\log x]$	[1]
	$y_1 = 2[\frac{1}{x+1} - \frac{1}{2}\times\frac{1}{x}] = \frac{x-1}{x(x+1)}$ (1)	[1]
	$y_2 = \frac{x(x+1)-(x-1)(2x+1)}{x^2(x+1)^2} = \frac{-x^2+2x+1}{x^2(x+1)^2}$	[1]
	$\Rightarrow x(x+1)^2 y_2 = \frac{-x^2+2x+1}{x} = \frac{2x-(x+1)(x-1)}{x} = 2-(x+1)^2 y_1$ (using (1))	[1]
	$\Rightarrow x(x+1)^2 y_2 + (x+1)^2 y_1 = 2.$ Hence, proved.	
16.	When y = 0, we have (x – 1) (x^2 + x + 1) (x – 2) = 0, i.e., x = 1 or 2.	[1/2]
	$\frac{dy}{dx} = x^3 - 1 + (x-2)3x^2 = 4x^3 - 6x^2 - 1$	[1/2]
	$(\frac{dy}{dx})_{(1,0)} = -3$	[1/2]
	$(\frac{dy}{dx})_{(2,0)} = 7$.	[1/2]
	The required equations of the tangents are y – 0 = -3 (x – 1) or, y = -3x + 3 and y – 0 = 7(x – 2) or, y = 7x – 14.	[2]
	OR	
	Domain f = $(-1,\infty)$ $f'(x) = \frac{-3}{1+x} + \frac{4}{(2+x)} + \frac{4}{(2+x)^2} = \frac{x(x+4)}{(1+x)(2+x)^2}$.	[1]
	$f'(x) = 0 \Rightarrow x = 0$ $[x \neq -4$ as $-4 \notin (-1,\infty)]$.	[1]
	In (-1, 0), $f'(x) = \frac{(-ve)(+ve)}{(+ve)(+ve)} = -ve$. Therefore, f is strictly decreasing in (-1, 0].	[1]
	In $(0,\infty)$, $f'(x) = +ve$. Therefore, f is strictly increasing in $[0,\infty)$.	[1]

17.	We have $C(x)=x^3-45x^2+600x, 10\le x\le 20$. For the time being we may assume that the function C(x) is continuous in [10, 20]. $C'(x)=3x^2-90x+600=3(x-10)(x-20)$ $C'(x)=0 \ if \ \ x=10 \ or \ \ x=20.$ But, 10, 20 $\notin(10,20)$. Therefore, the maximum or the minimum value will occur at the points. $C(10)=2500, C(20)=2000.$ Hence, the person must place the order for 20 trees and the least amount to be spent = Rs 2000. **Value: The person cares for a healthy environment despite being economically constrained.**	[1] [1] [1] [1]
18.	$\int\frac{\sec x}{1+\cos ecx}dx=\int\frac{\sin x}{\cos x(1+\sin x)}dx=\int\frac{\sin x\cos x}{(1+\sin x)^2(1-\sin x)}dx$ $=\int\frac{t}{(1+t)^2(1-t)}dt \ [\sin x=t\Rightarrow\cos xdx=dt]$ $\frac{t}{(1+t)^2(1-t)}=\frac{A}{1+t}+\frac{B}{(1+t)^2}+\frac{C}{1-t}\Rightarrow t=A(1+t)(1-t)+B(1-t)+C(1+t)^2$ (an identity) Put t = -1, - 1 = 2 B, i.e., B = -½. Put, t = 1, 1 = 4C, i.e., C = ¼. Put t = 0, 0 = A + B + C, which gives A = $\frac{1}{4}$. Therefore the required integral $=\frac{1}{4}\int\frac{1}{1+t}dt+\frac{-1}{2}\int\frac{1}{(1+t)^2}dt+\frac{1}{4}\int\frac{1}{(1-t)}dt$ $=\frac{1}{4}\log\lvert 1+t\rvert+\frac{-1}{2}\times\frac{-1}{1+t}-\frac{1}{4}\log\lvert 1-t\rvert+c$ $=\frac{1}{4}\log\lvert 1+\sin x\rvert+\frac{1}{2}\times\frac{1}{1+\sin x}-\frac{1}{4}\log\lvert 1-\sin x\rvert+c$	[1] [1+ 1/2] [1 + 1/2]
19.	The given differential equation is $ye^y dx=(y^3+2xe^y)dy, y(0)=1$ or, $\frac{ye^y}{(y^3+2xe^y)}=\frac{dy}{dx}$ $or, \frac{dx}{dy}+(-\frac{2}{y})x=\frac{y^2}{e^y}$, which is linear in x. I. F. = $e^{\int\frac{-2}{y}dy}=e^{-2\log y}=\frac{1}{y^2}$ Multiplying both sides by the I. F. and integrating, we get, $x\frac{1}{y^2}=\int e^{-y}dy$ $\Rightarrow x\frac{1}{y^2}=-e^{-y}+c\Rightarrow x=-y^2e^{-y}+cy^2$ (the general solution). When x = 0, y = 1. $0=-e^{-1}+c\Rightarrow c=\frac{1}{e}.$ Hence, the required particular solution is $x=-y^2e^{-y}+\frac{y^2}{e}.$ **OR**	[1] [1] [1/2] [1] [1/2]

	The given differential equation is $(x-y)dy=(x+2y)dx \; or, \dfrac{dy}{dx}=\dfrac{x+2y}{x-y}=\dfrac{1+2\frac{y}{x}}{1-\frac{y}{x}}=f(\frac{y}{x}),$ hence, homogeneous. Put y = v x $\Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$. The equation becomes $v+x\dfrac{dv}{dx}=\dfrac{1+2v}{1-v} \; or, \dfrac{1-v}{v^2+v+1}dv=\dfrac{dx}{x}$ $or, \dfrac{-1}{2}\times\dfrac{2v+1-3}{v^2+v+1}dv=\dfrac{dx}{x} \; or, [\dfrac{2v+1}{v^2+v+1}+\dfrac{-3}{v^2+2v\times\frac{1}{2}+(\frac{1}{2})^2+\frac{3}{4}}]dv=\dfrac{-2dx}{x}$ Integrating, we get $\int\dfrac{2v+1}{v^2+v+1}dv+\int\dfrac{-3}{(v+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}dv=\int\dfrac{-2dx}{x}$ or, $\log(v^2+v+1)-\dfrac{3\times 2}{\sqrt{3}}\tan^{-1}\dfrac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}}=-2\log\lvert x\rvert+c$ or, $\log(y^2+xy+x^2)-2\sqrt{3}\tan^{-1}\dfrac{2y+x}{\sqrt{3}x}=c$ (the general solution).	[1] [1] [2]
20.	$\vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{a}\times\vec{0}$ $\Rightarrow \vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{0}\Rightarrow \vec{a}\times\vec{b}=\vec{c}\times\vec{a}$ $\vec{b}\times(\vec{a}+\vec{b}+\vec{c})=\vec{b}\times\vec{0}$ $\Rightarrow \vec{b}\times\vec{c}=\vec{a}\times\vec{b} \; \therefore \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ $[\vec{a} \;\; \vec{b} \;\; \vec{c}]=\vec{a}\cdot(\vec{b}\times\vec{c})=\vec{a}\cdot(\vec{a}\times\vec{b})$ $=0$ [As the scalar triple product of three vectors is zero if any two of them are equal.]	[1] [1] [1/2] [1/2] [1/2] [1/2]
21.	General point on the first line is $(\lambda-2, 2\lambda+3, 4\lambda-1)$. General point on the second line is $(2\mu+1, 3\mu+2, 4\mu+3)$. Direction ratios of the required line are $\langle\lambda-3, 2\lambda+2, 4\lambda-2\rangle$. Direction ratios of the same line may be $\langle 2\mu, 3\mu+1, 4\mu+2\rangle$. Therefore, $\dfrac{\lambda-3}{2\mu}=\dfrac{2\lambda+2}{3\mu+1}=\dfrac{4\lambda-2}{4\mu+2}$ (1) $\Rightarrow \dfrac{\lambda-3}{2\mu}=\dfrac{2\lambda+2}{3\mu+1}=\dfrac{2\lambda-1}{2\mu+1}=k \; (say)$ $\Rightarrow \lambda-3=2\mu k, 2\lambda+2=(3\mu+1)k, \; 2\lambda-1=(2\mu+1)k$ $\Rightarrow \dfrac{\lambda-3}{2}=\mu k, 2\lambda+2=3\times\dfrac{\lambda-3}{2}+k, 2\lambda-1=\lambda-3+k$ $\Rightarrow k=\dfrac{4\lambda+4-3\lambda+9}{2}=\lambda+2\Rightarrow \lambda=9, \mu=\dfrac{3}{11}$, which satisfy (1). Therefore, the direction ratios of the required line are $\langle 6,20,34\rangle \; or, \langle 3,10,17\rangle$. Hence, the required equation of the line is $\dfrac{x-1}{3}=\dfrac{y-1}{10}=\dfrac{z-1}{17}$.	[1/2] [1/2] [1/2] [1/2] [1] [1/2] [1/2]

22. Let E_1 = Bag I is chosen, E_2 = Bag II is chosen, E_3 = Bag III is chosen, A = The two balls drawn from the chosen bag are white and red. [1]

$$P(E_1)=\frac{1}{3}=P(E_2)=P(E_3),$$

$$P(A|E_1)=\frac{1}{6}\times\frac{3}{6}\times 2, P(A|E_2)=\frac{2}{4}\times\frac{1}{4}\times 2, P(A|E_3)=\frac{4}{9}\times\frac{2}{9}\times 2.$$ [2]

By Bayes's Theorem, the required probability =

$$P(E_3|A)=\frac{P(E_3)\times P(A|E_3)}{\sum_{i=1}^{3}P(E_i)\times P(A|E_i)}=\frac{\frac{1}{3}\times\frac{4}{9}\times\frac{2}{9}\times 2}{\frac{1}{3}\times\frac{1}{6}\times\frac{3}{6}\times 2+\frac{1}{3}\times\frac{2}{4}\times\frac{1}{4}\times 2+\frac{1}{3}\times\frac{4}{9}\times\frac{2}{9}\times 2}=\frac{64}{199}.$$ [1]

23. Let X denotes the random variable. Then X = 0, 1, 2.

$$P(X=0)=\frac{{}^{16}c_2}{{}^{20}c_2}=\frac{60}{95}, P(X=1)=\frac{{}^{4}c_1\times{}^{16}c_1}{{}^{20}c_2}=\frac{32}{95}, P(X=2)=\frac{{}^{4}c_2}{{}^{20}c_2}=\frac{3}{95}.$$ [1 + 1/2]

x_i	p_i	x_ip_i	$x_i^2p_i$
0	60/95	0	0
1	32/95	32/95	32/95
2	3/95	6/95	12/95
total		38/95	44/95

[1/2]

Mean = $\sum_{i=1}^{3}x_ip_i=\frac{38}{95}=\frac{2}{5}$ [1/2]

Variance = $\sum_{i=1}^{3}x_i^2p_i-(\sum_{i=1}^{3}x_ip_i)^2=\frac{44}{95}-\frac{4}{25}=\frac{144}{475}.$ [1 +1/2]

Section D

24. $f\circ g:\mathbb{R}\to\mathbb{R}$ defined by $f\circ g(x)=f(g(x))=f(x^3+5)=2(x^3+5)-3=2x^3+7$ [1]

Let $x_1, x_2\in\mathbb{R}(D_{f\circ g})$ such that

$f\circ g(x_1)=f\circ g(x_2)\Rightarrow 2x_1^{\,3}+7=2x_2^{\,3}+7\Rightarrow x_1^{\,3}=x_2^{\,3}\Rightarrow x_1=x_2$. Hence, $f\circ g$ is one-one. [1]

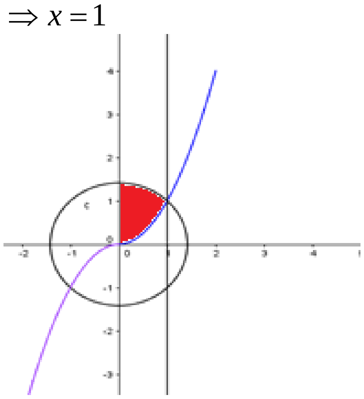
Let $y\in\mathbb{R}(Codomain_{f\circ g})$. Then for any x $f\circ g$ (x) = y if 2 x^3 + 7 = y, i.e., if, 2 x^3 = y – 7, i.e., x = $\sqrt[3]{\frac{y-7}{2}}$, which $\in\mathbb{R}(D_{f\circ g})$. Hence, for every $y\in\mathbb{R}(Codomain_{f\circ g}),\exists\sqrt[3]{\frac{y-7}{2}}\in\mathbb{R}(D_{f\circ g})$ [2]

such that $f\circ g(\sqrt[3]{\frac{y-7}{2}})=y$. Hence,

$f\circ g$ is onto.

Since, $f\circ g$ is both one-one and onto, it is invertible. [1/2]

	$(f\circ g)^{-1}:\mathbb{R}\to\mathbb{R}$ defined by $(f\circ g)^{-1}(x)=\sqrt[3]{\dfrac{x-7}{2}}$ $(f\circ g)^{-1}(9)=\sqrt[3]{\dfrac{9-7}{2}}=1$. **OR** Let $a,b\in\mathbb{R}$ such that a = 0, b $\neq$ 0. Then $a*b=\lvert a\rvert+b=\lvert 0\rvert+b=b, b*a=b, \therefore a*b=b*a$ Let $a,b\in\mathbb{R}$ such that a $\neq$ 0, b = 0. Then $a*b=a, b*a=\lvert b\rvert+a=\lvert 0\rvert+a=a, \therefore a*b=b*a$ Let $a,b\in\mathbb{R}$ such that a = 0, b = 0. Then $a*b=a=0, b*a=b=0, \therefore a*b=b*a$. Now we need to check whether $*$ is commutative. One more case is needed to be examined. Let $a,b\in\mathbb{R}$ such that a $\neq$ 0, b $\neq$ 0. Then $a*b=\lvert a\rvert+b, b*a=\lvert b\rvert+a$ and $a*b$ may not be equal to $b*a$, e.g., (-1)$*$2=3, 2$*$ (-1) = 1, hence, (-1)$*$2 $\neq$ 2$*$ (-1). Thus $*$ is not commutative. The element $e\in\mathbb{R}$ will be the identity element for $*$ if a$*$ e = e $*$ a = a for all $a\in\mathbb{R}$. $a*e=a$ provided e = 0 and $e*a=a$ provided e = 0 (As $0*0=0$ and $0*a=\lvert 0\rvert+a=a$ for a $\neq$0). Hence, 0 is the identity element for $*$.	[1] [1/2] [1] [1] [1] [1] [2]
25.	$\lvert A\rvert=3(3-6)+(-2)(-12-14)+1(12+7)=62\neq 0$. Hence, A^{-1} exists. Let c_{ij} represent the cofactor of (i, j)th element of A. Then, $c_{11}=-3, c_{12}=26, c_{13}=19, c_{21}=9, c_{22}=-16, c_{23}=5, c_{31}=5, c_{32}=-2, c_{33}=-11.$ adjA = $\begin{bmatrix}-3 & 9 & 5\\ 26 & -16 & -2\\ 19 & 5 & -11\end{bmatrix}$ $A^{-1}=\dfrac{1}{62}\begin{bmatrix}-3 & 9 & 5\\ 26 & -16 & -2\\ 19 & 5 & -11\end{bmatrix}$ The given system of equations is equivalent to the matrix equation $A'X=B, \text{ where } X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}14\\ 4\\ 0\end{bmatrix}.$ $\Rightarrow X=(A')^{-1}B=(A^{-1})'B$ $=\dfrac{1}{62}\begin{bmatrix}-3 & 26 & 19\\ 9 & -16 & 5\\ 5 & -2 & -11\end{bmatrix}\begin{bmatrix}14\\ 4\\ 0\end{bmatrix}=\dfrac{1}{62}\begin{bmatrix}62\\ 62\\ 62\end{bmatrix}=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$ Hence, x = 1, y = 1, z = 1 **OR**	[1] [2] [1] [2]

	$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \; (R_1 \leftrightarrow R_2)$	[1]
	$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \; (R_2 \to R_2 - 2R_1)$	[1]
	$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -2 & 4 & 1 \end{bmatrix} A \; (R_3 \to R_3 - 2R_2)$	[1]
	$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix} A \; (R_2 \to R_2 + R_3, R_1 \to R_1 - R_3)$	[1]
	$\Rightarrow A^{-1} = \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix}$	[1]
	$X = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$	[1]
26.	$y = x\lvert x \rvert = \begin{cases} x^2, x \geq 0 \\ -x^2, x < 0 \end{cases}.$	[1]
	Solving $y = x^2, x^2 + y^2 = 2$ simultaneously, $y + y^2 - 2 = 0 \Rightarrow (y+2)(y-1) = 0 \Rightarrow y = 1$ ($y = x^2$ lies in quadrant I). $\Rightarrow x = 1$	[1/2]
		[1]
	The required area = the shaded area = $\int_0^1 (\sqrt{2-x^2} - x^2)dx$	[2]
	$= \frac{1}{2}[x\sqrt{2-x^2} + 2\sin^{-1}\frac{x}{\sqrt{2}}]_0^1 - \frac{1}{3}[x^3]_0^1 = (\frac{1}{6} + \frac{\pi}{4})$ sq units.	[1+1/2]

27.	The given definite integral = I = $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x}{2-\cos 2x}dx+\frac{\pi}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2-\cos 2x}dx$ $f(x)=\frac{x}{2-\cos 2x},\ f(-x)=\frac{-x}{2-\cos 2x}=-f(x).$ Hence, f is odd. Therefore, $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x}{2-\cos 2x}dx=0$ $g(x)=\frac{1}{2-\cos 2x},\ g(-x)=\frac{1}{2-\cos 2x}=g(x).$ Hence, g is even. Thus $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2-\cos 2x}dx=2\int_{0}^{\frac{\pi}{4}} \frac{1}{2-\cos 2x}dx.$ Hence, I = $\frac{\pi}{2}\int_{0}^{\frac{\pi}{4}} \frac{1}{2-\cos 2x}dx=\frac{\pi}{2}\int_{0}^{\frac{\pi}{4}} \frac{1}{1+2\sin^2 x}dx$ $=\frac{\pi}{2}\int_{0}^{\frac{\pi}{4}} \frac{\sec^2 x}{1+\tan^2 x+2\tan^2 x}dx=\frac{\pi}{2}\int_{0}^{\frac{\pi}{4}} \frac{\sec^2 x}{1+3\tan^2 x}dx=\frac{\pi}{2}\int_{0}^{1} \frac{1}{1+3t^2}dt\ [\tan x=t\Rightarrow \sec^2 xdx=dt]$ $=\frac{\pi}{2}\times\frac{1}{3}\int_{0}^{1} \frac{1}{(\frac{1}{\sqrt{3}})^2+t^2}dt\ =\frac{\pi}{6}\sqrt{3}[\tan^{-1}\sqrt{3}t]_0^1$ $=\frac{\pi}{6}\sqrt{3}[\frac{\pi}{3}]=\frac{\sqrt{3}\pi^2}{18}$ **OR** Let f(x) = $3x^2-2x+4$. Then the given definite integral = $\int_{-2}^{2} f(x)dx=\lim_{\substack{n\to\infty \\ h\to 0}} h\sum_{r=1}^{n} f(-2+rh),\ where\ nh=4.$ $f(-2+rh)=3(-2+rh)^2-2(-2+rh)+4=3r^2h^2-14rh+20$ $\sum_{r=1}^{n} f(-2+rh)=3h^2\sum_{r=1}^{n} r^2-14h\sum_{r=1}^{n} r+20n=3h^2\times\frac{n(n+1)(2n+1)}{6}-14h\times\frac{n(n+1)}{2}+20n$ $\int_{-2}^{2} f(x)dx=\lim_{\substack{n\to\infty \\ h\to 0}} h[3h^2\times\frac{n(n+1)(2n+1)}{6}-14h\times\frac{n(n+1)}{2}+20n]$ $=\lim_{\substack{n\to\infty \\ h\to 0}}[3\times\frac{nh(nh+h)(2nh+h)}{6}-14\times\frac{nh(nh+h)}{2}+20nh]$ $=\lim_{h\to 0}[\frac{4(4+h)(8+h)}{2}-7\times 4(4+h)+20\times 4]=32$	[1] [1] [2] [2] [1] [1] [2 + 1/2] [1 + 1/2]

28. The general point on the given line $\frac{x-1}{1}=\frac{y-2}{3}=\frac{z+1}{-9}$ [1]

is $(\lambda+1, 3\lambda+2, -9\lambda-1)$. [1]

The direction ratios of the line parallel to the plane x – y +2 z -3 =0 intersecting the given line and passing through the point (-2, 3, -4) are $\langle \lambda+3, 3\lambda-1, -9\lambda+3\rangle$ [1]

and $(\lambda+3)1+(3\lambda-1)(-1)+(-9\lambda+3)2=0 \Rightarrow \lambda=\frac{1}{2}$. [1]

The point of intersection is $(\frac{3}{2},\frac{7}{2},\frac{-11}{2})$. [1]

The required distance = $\sqrt{(\frac{3}{2}+2)^2+(\frac{7}{2}-3)^2+(\frac{-11}{2}+4)^2}=\frac{\sqrt{59}}{2}$ unit. [1]

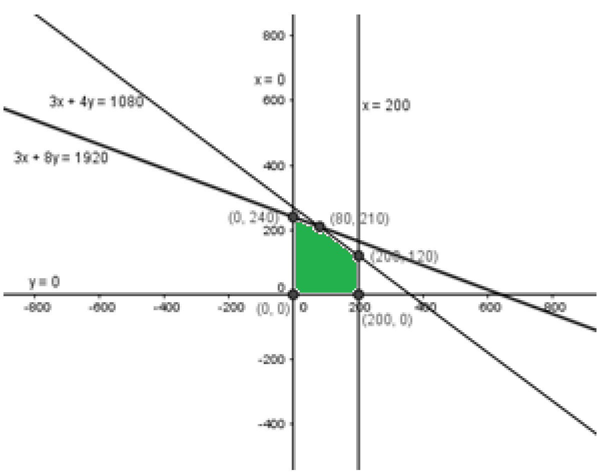
29. Let x = the number of units of Product 1 to be produced daily

y = the number of units of Product 2 to be produced daily

To maximize P = (9 – 1.2)x + (8 – 0.9)y = 7.8x + 7.1y [1]

subject to the constraints:

$\frac{x}{4}+\frac{y}{3}\le 90, or\ 3x+4y\le 1080, \frac{x}{8}+\frac{y}{3}\le 80, or\ 3x+8y\le 1920, x\le 200, x\ge 0, y\ge 0.$ [2]



[2]

At the point	P
(0, 0)	0
(200, 120)	2412
(0, 240)	1704
(200, 0)	1560
(80, 210)	2115

[1]

The maximum profit = Rs. 2412.