राष्ट्रीय नवोदय नेतृत्व संस्थान

नवोदय विद्यालय समिति,मुख्यालय,नोएडा

NATIONAL NAVODAYA LEADERSHIP INSTITUTE, (NVS H.Q., NOIDA)

CONTENT DEVELOPMENT WORKSHOP

(From 11-09-2018 to 20-09-2018)

SUBJECT: MATHEMATICS
CLASS: XI

SUPPORT MATERIAL

(UNIT WISE ACTIVITIES)

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CONTENT

UNIT - I

TOPIC - SETS AND FUNCTIONS

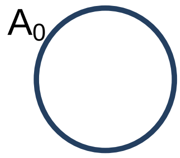
Activity-1

AIM:-To verify that the number of subsets of a set is 2^n,where n is the number of elements in the set.

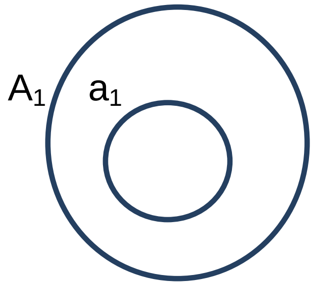
MATERIAL REQUIRED:-Drawing sheet, scissors, Geometry box, colour sketch pen.

DEMONSTRATION:-

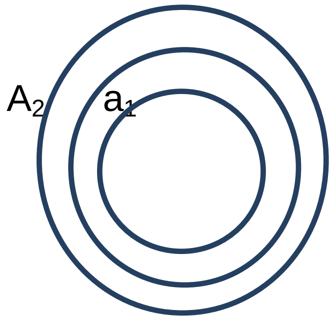
(1) Take an empty set A_0 which have no element., (fig.1.1)



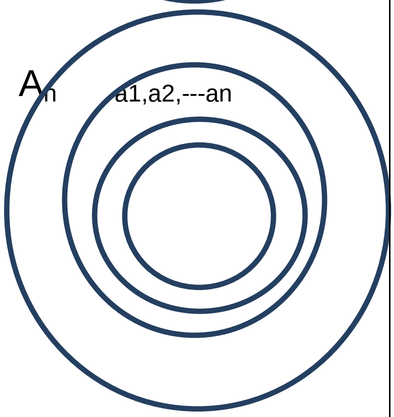
(2)Take a set A_1 which has 1 element, say a_1 (fig1.2)



(3) Take a set A_2 which has two elements, say a1,a2 (fig1.3)a_2



Take a set A_n which has n element



CALCULATION:-

(i)Here the number of subsets of set $A_0 = 1 = 2^0$

(ii)Here subset of set $A_1$are $\emptyset$, $\{a_1\}$ hence number of subsets $=2=2^1$

(iii) Here the subset of set A_2 are ∅, $\{a_1\}$,$\{a_2\}$,$\{a_1,a_2\}$

Hence the number of subset of set $A_2 = 4 = 2^2$

Here the number of subsets of An = ∅,$\{a_1\}$, $\{a_1,a_2\}$, …$\{a_1,a_2,a_3{----}a_n\}$= 2^n

CONCLUSION:-

Thus by the above steps we have verified that 2^n will be the number of subsets of a given set.

Activity-2

AIM:- To verify the number of elements in a power set of a set A is 2^n, where n is the number of elements in set A ie $A = \{a_1, a_2, a_3, ---, a_n\}$

MATERIAL REQUIRED:- Drawing sheet, scissors, Geometry box, sketch pen

PROCEDURE:-

(i) Take a box having no element

(ii) Take a box having one element

(iii) Take a box having two elements

(iv)Take a box having n elements

DEMONSTRATION:-

(i) ∅ represent all the selection when taking no element out of n elements, hence number of selection when taking no element out of n elements $= {}^nc_0 = 1 = \emptyset$

(ii) $\{a_1\}, \{a_2\}, ----\{a_n\}$ represent all the selection when taking one-one element out

of n elements, hence number of selection taking one-one element out of n

elements $= {}^nc_1 = n = \{a_1\}, \{a_2\}, ----\{a_n\}$

(ii) $\{a_1, a_2\}, \{a_1, a_3\}$----upto n elements represent all the selection when taking two-two elements out of n ,hence number of selection when taking two-two elements out of n elements $= {}^nc_2 = \{a_1, a_2\}, \{a_1, a_3\}$----upto n elements

----

(iii) $\{a_1, a_2, a_3 ---- a_n\}$ represent all the selection when taking n elements out of n , hence number of selection when taking n elements out of n elements $= {}^nc_n = 1 = \{a_1, a_2, a_3, ------- a_n\}$

OBSERVATION TABLE:-

No. of elements taking r at a time	Different subsets
0	${}^nc_0= 2^0=1$
1	${}^nc_1= 2^1=2$
2	${}^nc_2= 2^2=4$
3	${}^nc_3= 2^3=8$
----	----
N	${}^nc_n= 2^n$

Sum = ${}^nc_0+{}^nc_1+{}^nc_2+{}^nc_3{}^+$-----$+{}^nc_n = \sum_{k=0}^{n} {}^nC_k$

RESULT:-

Using $(a+b)^n = {}^nc_0a^n+{}^nc_1a^{n-1} b+$----$+{}^nc_nb^n$, where n is positive

Integer Put a = b = 1then ${}^nc_0+{}^nc_1+{}^nc_2+{}^nc_3{}^+$-----$+{}^nc_n =2^n$

Or $$\sum_{k=0}^{n} {}^nC_k = 2^n$$

NOTE:- This activity can be conducted after completion of binomial theorem.

Activity-3

AIM:-To represent set operations using Venn diagrams.

MATERIAL REQUIRED:- Drawing sheet, scissors, Geometry box, sketch pen

METHOD OF CONSTRUCTION:-

(a)Cut rectangular strips from a sheet of paper and paste them on a hard board .write the symbol U in the left top corner of the each rectangle.

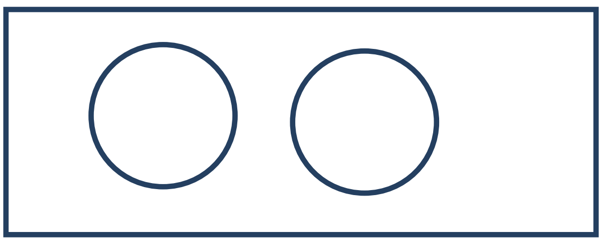
(b) Draw two circles A and B inside each of the rectangular strips.

DEMONSTRATION:-

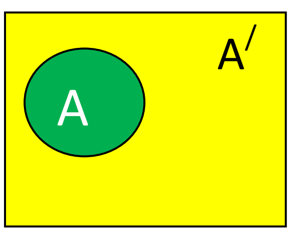
(i) U denotes the Universal set represented by a rectangle.

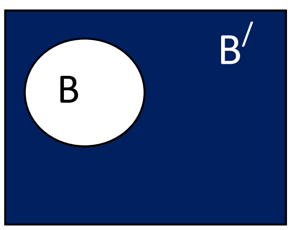
(ii) Circles A and B represent the subsets of the universal set U.

U A B

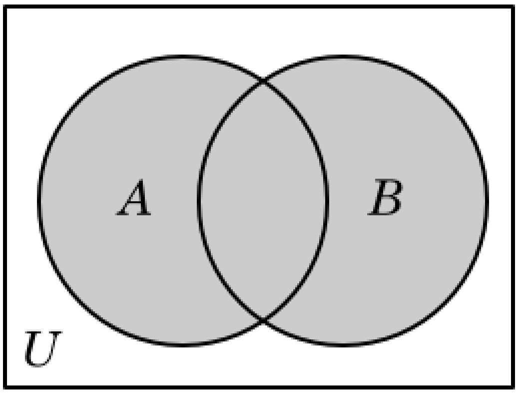


(iii) A denotes the complement of the set A, which is yellow coloured and B´ denotes the complement of the set B, which is blue colored shade .

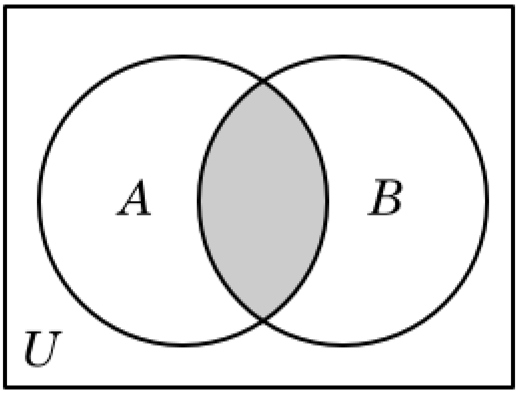




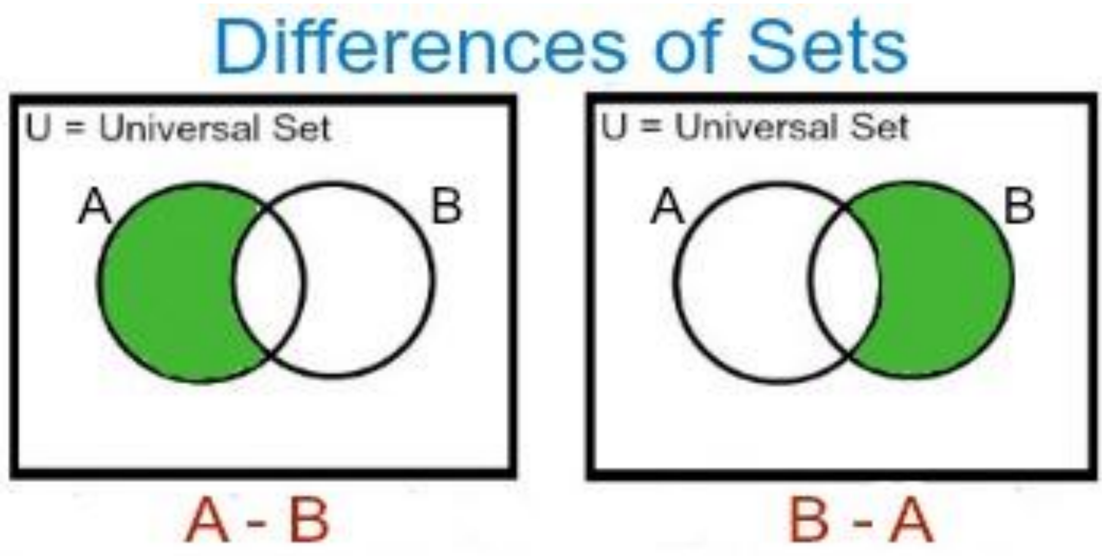
(iv) Shaded portion in the following fig. represents A∪B



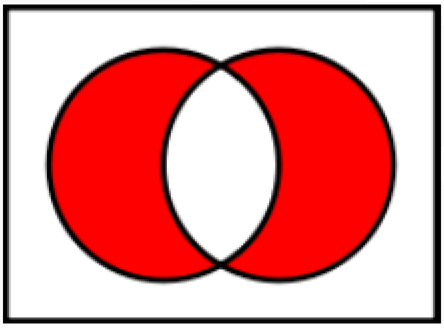
(v) Shaded region denote A∩B



(vi)) Shaded region denote A-B and B-A



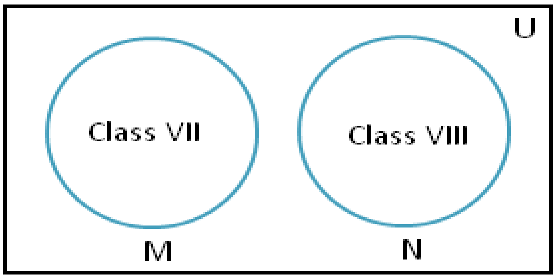
(vii)Shaded region denote symmetric difference of A and B



Symmetric Difference of Sets

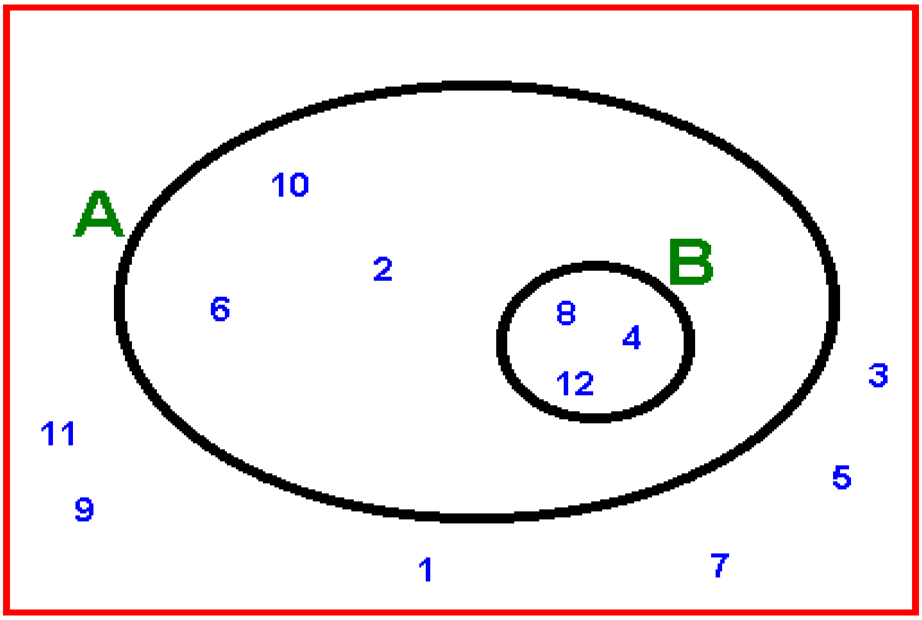
$A \triangle B$

(viii) Disjoint sets

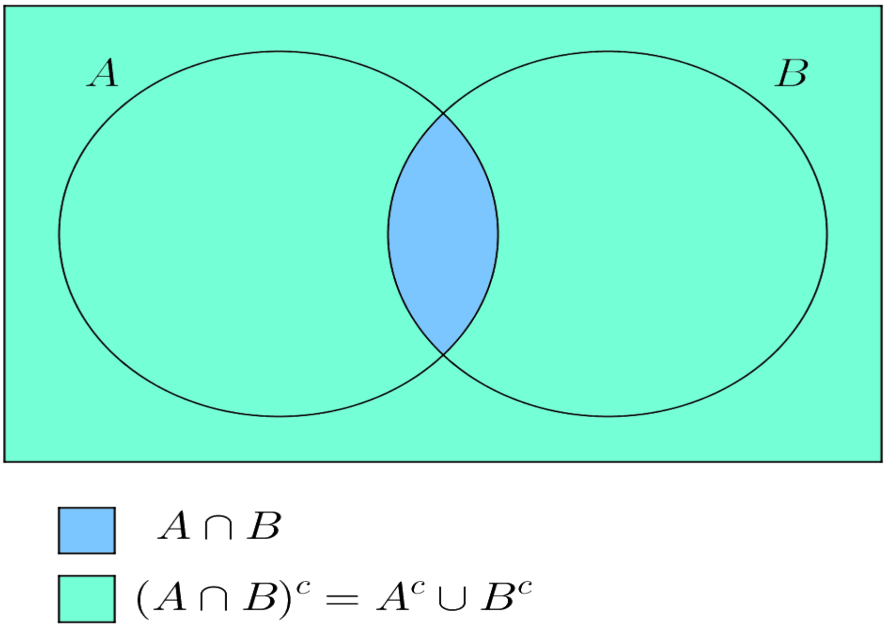


Disjoint set

(ix)Subsets $B \subset A$



(x) De Morgan law



<u>CONCLUSION:-</u>Thus we have visualized various operations on Sets by using Venn diagram.

TOPIC- RELATIONS AND FUNCTIONS

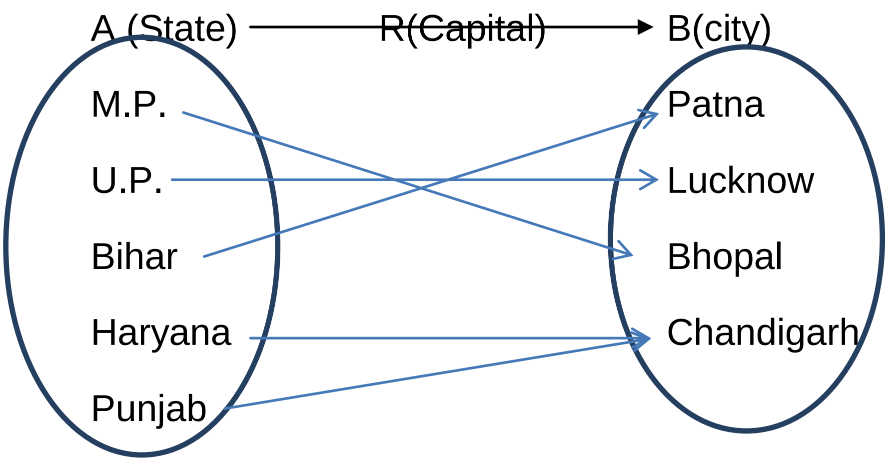
Activity-4

AIM:- To show the relation from a set A to set B.

MATERIAL REQUIRED:- Drawing sheet, scissors, Geometry box, Sketch Pen.

DEMONSTRATION:- Let a set A = {M.P., U.P., Bihar, Haryana, Punjab} and Set B= {Patna, Lucknow, Bhopal, Chandigarh }

Now we can show the relation from set A to set B by following diagram



Therefore the relation R is a subset of AXB as following

R = {(M.P., Bhopal),(U.P.,Lucknow),(Bihar,Patna),(Haryana,Chandigarh), (Punjab,Chandigarh)}

CONCLUSION:- Hence we can understand the relation from set A to set B

Relation R ={ (x,y): $x \in$state,$y \in$capital}

Domain = {M.P., U.P., Bihar, Haryana, Punjab} and

Range = {Patna, Lucknow, Bhopal ,Chandigarh }

Activity-5

Aim:- To verify that for two sets X and Y, number of ordered Pairs= p×q. and the total number of Relations from X to Y is $2^{p \times q}$, where n(X)=p and n(Y)=q.

Material Required:- Chart Paper ,sketch Pens of different colours.

Procedure:-

1.Drawa set X_1on chart paper and mark a red dot as an element a_1.

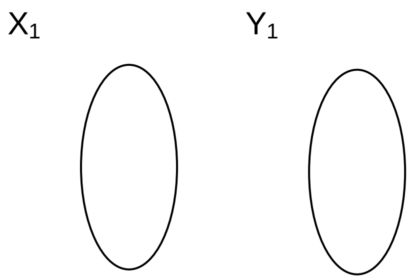
2.Draw another set on the same chart paper and name it set Y_1 and Mark one green point within it as an element b_1.Hence $n(X_1)=1$ and $n(Y_1)=1$.

3.Draw an arrow from X_1 to Y_1 .which connect element of X_1 to element of Y_1.

1. Take a set X_2 which has two elements (say) a_1,a_2 and take another set Y_2 which has one element say b_1.Hence $n(X_2)=2$ and $n(Y_2)=1$.
2. Take a set X_3 which has three elements say a_1,a_2,a_3 and take another set Y_3 which has two elements say b_1,b_2.Hence $n(X_3)=3$ and $n(Y_3)=2$.

Demonstration:-

1.Represent all the possible group of elements set X_1 to the element of set Y_1 in the shown figure.

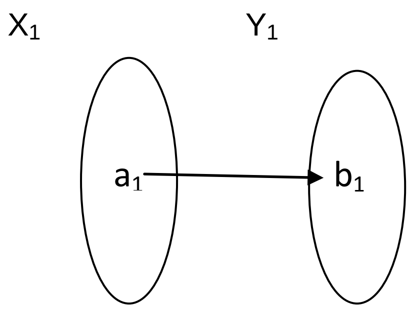


Number of ordered Pair= { }, such that, $n(X_1 \times Y_1)=n(X_1)\times n(Y_1)=p\times q=0\times 0=0$

Total number of Relations $= 2^{p\times q} = 2^{0\times 0}=1$.

This Relation is Void Relation.

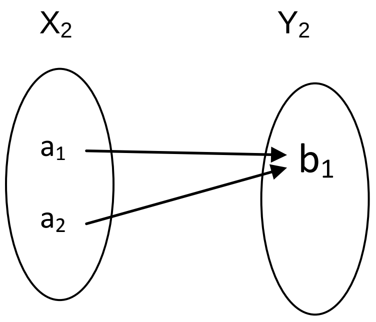
2.Represent all the possible group of elements set X_1 to the element of set Y_1 in the shown figure.



Number of ordered Pair={ (a_1, b_1)} .such that, $n(X_1 \times Y_1) = n(X_1) \times n(Y_1) = p \times q = 1 \times 1 = 1$

Total number of Relations $= 2^{p \times q} = 2^{1 \times 1} = 2$

2. Represent all the possible group of elements set X_2 to the element of set Y_2 in the shown figure.

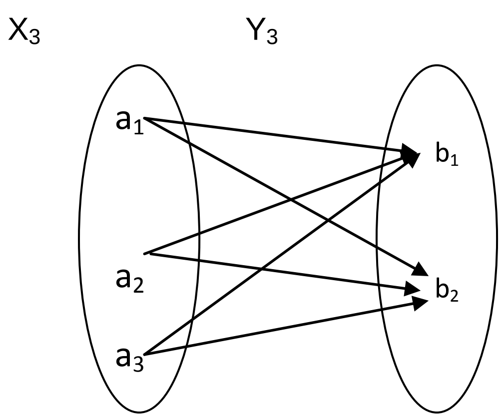


Number of ordered Pair ={$(a_1, b_1),(a_2,b_1)$ }. such that

$n(X_2 \times Y_2) = n(X_2) \times n(Y_2) = p \times q = 2 \times 1 = 2$

Total number of Relations $= 2^{p \times q} = 2^{2 \times 1} = 4$

3. Represent all the possible group of elements set X_3 to the element of set Y_3 in the shown figure.



Number of ordered Pair ={$(a_1, b_1),(a_1,b_2), (a_2,b_1), (a_2,b_2),(a_3,b_1),(a_3,b_2)$ }

.s.t, $n(X_3 \times Y_3) = n(X_3) \times n(Y_3) = p \times q = 3 \times 2 = 6$

Total number of Relations $= 2^{3 \times 2} = 2^6 = 64$.

Observation Table:-

No. of elements in set X	No. of elements in set Y	No. of ordered pairs	No. of total Relations
1	1	1	$2^{1\times1}=2$
2	1	2	$2^{2X1}=4$
3	2	6	$2^{3X2}=64$
3	3	9	$2^{3X3}=512$
4	2	-	-
4	3	-	-
4	4	-	-
0	4	-	-
4	0	-	-
6	2	-	-

Conclusion:-

1.The number of ordered pairs of two sets X and Y are $p\times q$ where $n(X)=p$ and $n(Y)=q$.

2.The number of total possible Relations for two sets X and Y are $2^{p\times q}$.where $n(X)=p$ and $n(Y)=q$

Activity-6

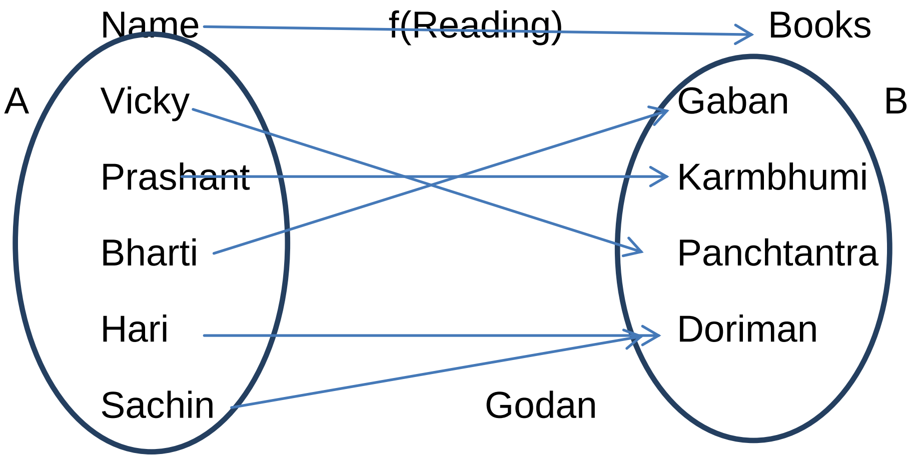
AIM:- To verify a function from the given relation from a set A to set B.

MATERIAL REQUIRED:- Drawing sheet, Scissors, Geometry box, Sketch Pen

DEMONSTRATION:- let a set A = {Vicky, Prashant, Bharti, Hari, Sachin} and

Set B={Gaban, Karmbhumi, Panchtantra ,Doriman, Godan}

Now we can show the function from set A to set B by following diagram



Therefore the function f is a subset of A X B as following

f={(Vicky,Panchtantra),(Prashant,Karmbhumi),(Bharti,Gaban),(Hari,Doriman),(Sachin ,Doriman)}

CONCLUSION:- Hence we can understand the function(Reading books) from set A to set B

Function f = { (x,y):   x∈ Name of student, y∈ Name of book)}

Domain = {Vicky, Prashant, Bharti, Hari, Sachin}

Co domain   = {Gaban, Karmbhumi, Panchtantra, Doriman, Godan} and

Range = {Gaban,Karmbhumi,Panchtantra,Doriman}

UNIT II

TOPIC : TRIGONOMETRIC FUNCTIONS

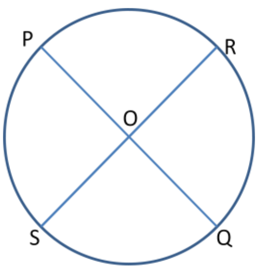
Activity – 7

Aim:- To verify the relation between the degree measure and the radian measure of an angle.

Material Required:- Geometry box, Protractor, Thread, Sketch pen, Cardboard, White paper.

Procedure:-

1. Cut a circle of suitable radius from chart paper and paste on white paper.
2. Draw two different diameters PQ and RS of the circle.
3. OP = OR = OS = OQ = radius.

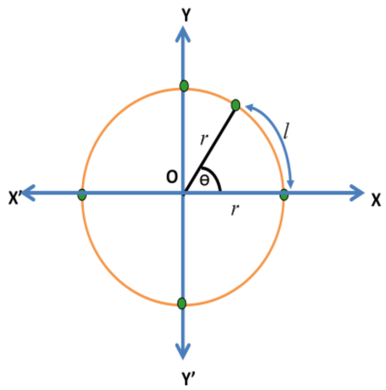


Demonstration:-

1. Let the radius of the circle be *r* and *l* be an arc subtending an angle θ at the centre O, such that $\theta = \frac{l}{r}$
2. If Degree measure of $\theta = \frac{l}{2\pi r} \times 360^0$

   then $\frac{l}{r} = \frac{l}{2\pi r} \times 360^0$

   1 radian = $\frac{180}{\pi}$ degrees = 57.27 degrees.



Observation:-

1. Using thread, measure arc lengths RP, SQ and record them in the table given below:

S.No.	Arc	Length of arc(l)	Radius of circle(r)	Radian Measure
1	RP			$angle\ ROP = \frac{arc\ RP}{r}$
2	SQ			$angle\ SOQ = \frac{arc\ SQ}{r}$

2. Using protractor, measure the angle in degrees and complete the table.

Angle	**Degree measure**	**Radian Measure**	**Ratio =** $\frac{\textit{Degree measure}}{\textit{Radian measure}}$
$angle\ ROP$			
$angle\ SOQ$			

3. The value of one radian is equal to ________ degrees.

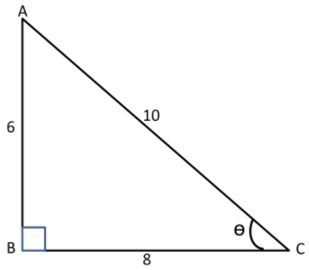
<u>Activity – 8</u>

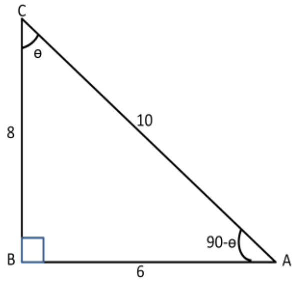
<u>Aim:-</u> To understand the relation between trigonometric ratios of complementary angles.

<u>Materials Required:-</u> Coloured chart paper, Sketch Pen, Ruler, Glue, Scissor, Geometry box etc.

<u>Procedure:-</u>

1. Cut a right angled triangle from coloured chart paper with sides 6cm, 8m and 10cm.
2. Paste it on white sheet.
3. Name the triangle as ΔABC right angled at B.
4. Mark $\angle C$ as ɵ.
5. Calculate all the trigonometric ratios for angle ɵ.
6. Cut another right angled triangle from coloured chart paper of same dimensions and paste it as shown in fig.
7. In this fig. the value of $\angle A$ is 90^0 – ɵ.
8. Now calculate all the trigonometric ratios for angle 90^0 – ɵ.





<u>Observation:-</u>

S. No.	T- ratios for angle ɵ		T- ratios for angle 90^0 – ɵ	
1	*sin ɵ*		*sin (90^0 – ɵ)*	
2	*cos ɵ*		*cos (90^0 – ɵ)*	
3	*tan ɵ*		*tan (90^0 – ɵ)*	
4	*cot ɵ*		*cot (90^0 – ɵ)*	
5	*sec ɵ*		*sec (90^0 – ɵ)*	
6	*cosec ɵ*		*cosec (90^0 – ɵ)*	

Conclusion:-

sin $(90^0 - \theta) =$ ***cos*** θ
cos $(90^0 - \theta) =$ ***sin*** θ
tan $(90^0 - \theta) =$ ***cot*** θ
cot $(90^0 - \theta) =$ ***tan*** θ
sec $(90^0 - \theta) =$ ***cosec*** θ
cosec $(90^0 - \theta) =$ ***sec*** θ

Activity – 9

Aim:- To understand the signs of trigonometric functions in different quadrants.

Basic Concepts:- Any point lying on unit circle has coordinates of the form (cosx, sinx), where x is the angle in radian subtended by the corresponding arc at the centre.

Materials Required:- Graph papers, Sketch Pen, Ruler, Glue, Scissor, Geometry box etc.

Procedure:-

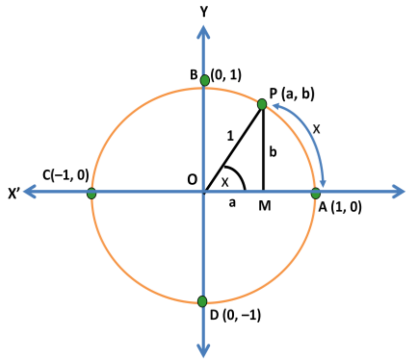
1. Draw two mutually perpendicular lines on graph paper and mark them as x-axis and y-axis.
2. With origin as centre, draw a unit circle intersecting the axes at points A, B, C and D respectively.
3. Take any point P(a,b) on the unit circle such that $\angle AOP = x$ radian.

Case I: When the point P(a,b) lies in first quadrant ($0 < x < \frac{\pi}{2}$)

Since P(a, b) lies in I^{st} quadrant

so ‘a’ and ‘b’ are both positive.

Thus, *cos x = a = +ve & sin x = b =+ve.*

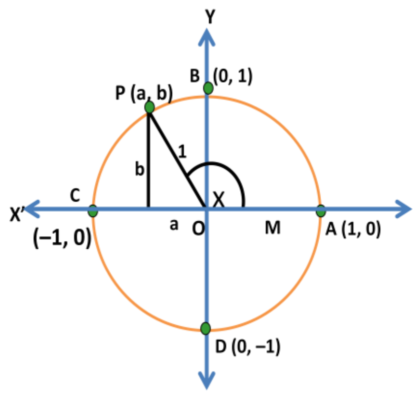


Case II: When the point P(a,b) lies in II^{nd} quadrant ($\frac{\pi}{2} < x < \pi$)

Since P(a, b) lies in II^{nd} quadrant

so ‘a’ is negative and ‘b’ is positive.

Thus, *cos x = a = –ve & sin x = b =+ve.*

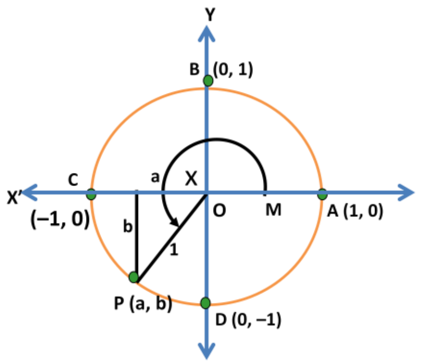


Case III: When the point P(a,b) lies in IIIrd quadrant (π < x < $\frac{3\pi}{2}$)

Since P(a, b) lies in IIIrd quadrant

so ‘a’ is negative and ‘b’ is positive.

Thus, *cos x = a = –ve & sin x = b = –ve.*

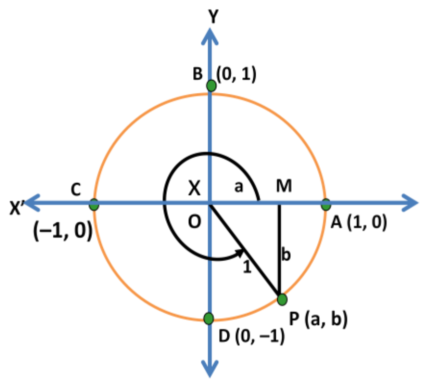


Case IV: When the point P(a,b) lies in IVth quadrant ($\frac{3\pi}{2}$ < x < 2π)

Since P(a, b) lies in IVth quadrant

so ‘a’ is positive and ‘b’ is negative

Thus, *cos x = a = +ve & sin x = b = –ve.*



Observation:-

	$I(a, b)$	$II(-a, b)$	$III(-a, -b)$	$IV(a, -b)$
$sinx = b$	$+ve$	$+ve$	$-ve$	$-ve$
$cos\ x = a$	$+ve$	$-ve$	$-ve$	$+ve$
$tanx = \frac{b}{a}, a \neq 0$	$\frac{+ve}{+ve} = +ve$	$\frac{+ve}{-ve} = -ve$	$\frac{-ve}{-ve} = +ve$	$\frac{-ve}{+ve} = -ve$
$cot\ x = \frac{a}{b}, b \neq 0$	$\frac{+ve}{+ve} = +ve$	$\frac{-ve}{+ve} = -ve$	$\frac{-ve}{-ve} = +ve$	$\frac{+ve}{-ve} = -ve$
$secx = \frac{1}{a}, a \neq 0$	$\frac{1}{+ve} = +ve$	$\frac{1}{-ve} = -ve$	$\frac{1}{-ve} = -ve$	$\frac{1}{+ve} = +ve$
$cosec\ x = \frac{1}{b}, b \neq 0$	$\frac{1}{+ve} = +ve$	$\frac{1}{+ve} = +ve$	$\frac{1}{-ve} = -ve$	$\frac{1}{-ve} = -ve$

Conclusion:-

1. In Ist quadrant all trigonometric functions are +ve.
2. In IInd quadrant sine and cosecant are +ve and others are –ve.
3. In IIIrd quadrant tangent and cotangent are +ve and others are –ve.
4. In IVth quadrant cosine and secant are +ve and others are –ve.

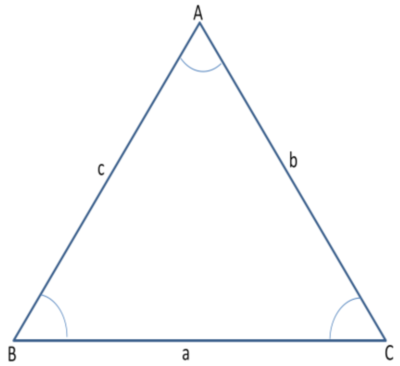
Activity – 10

Aim:- To verify the sine formulae.

Materials Required:- Coloured chart paper, Sketch Pen, Ruler, Glue, Scissor, Geometry box etc.

Procedure:-

1. Draw a triangle ABC on coloured chart paper such that BC = 7cm, $\angle B = 60^0$ & $\angle C = 45^0$.
2. Measure $\angle A$ and sides AB & AC of the triangle ABC.
3. Cut the triangle ABC and paste it on white sheet.
4. Let BC = a = 7cm, AC = b = ______ & AB = c = ______.



Observation:-

S.No.	Value of sine	Sides of triangle	Ratio
1.	*sin A=*	a =	$\frac{\sin A}{a} =$
2.	*sin B=*	b =	$\frac{\sin B}{b} =$
3.	*sin C=*	c =	$\frac{\sin C}{c} =$

Conclusion:-

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

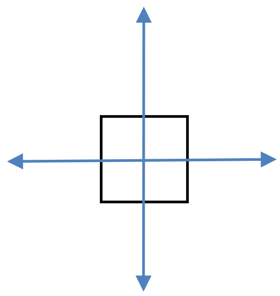
CHAPTER 4: PRINCIPLE OF MATHEMATICAL INDUCTION

Activity – 11

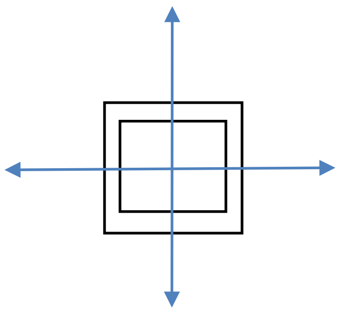
Aim:- To understand the Principle of mathematical induction.

Materials Required:- Graph sheet, Sketch Pen, Ruler, Glue, Scissor, Geometry box etc.

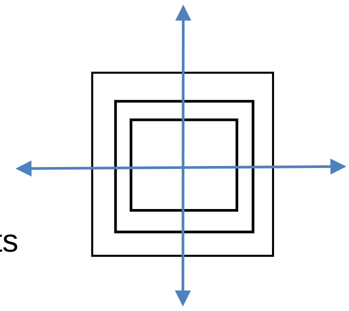
Procedure:-



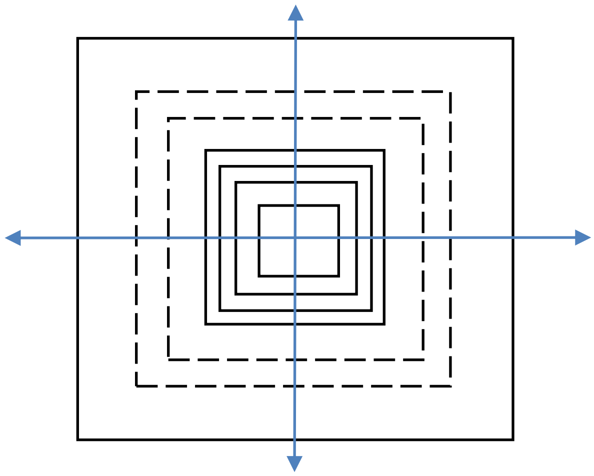
1. Draw a square of side 1 unit.
2. Find the perimeter of the square,
   P_1= 4 x side = 4 x 1
3. Take another square of side 2 units.
4. Find the perimeter of this square,
   P_2= 4 x side = 4 x 2



5. Take another square of side 3 units.
6. Find the perimeter of this square,
   P_3= 4 x side = 4 x 3
7. Take another square of side 4 units.
8. Find the perimeter of this square,
   P_4= 4 x side = 4 x 4



9. Take this process up to n times.
10. Find the perimeter of the square with side n units
    P_n= 4 x side = 4 x n



Observation(Worksheet):-

Square	Side of Square	Perimeter	Number of squares	Sum of Perimeters
1^{st} Square	1 unit	4 x 1 = 4units	01	$S_1=P_1= 4$
2^{nd} Square	2 units	4 x 2 = 8 units	02	$S_2= P_1 + P_2 = 4 + 8 =12$
3^{rd} Square	3 units	4 x 3 = 12 units	03	$S_3= P_1 + P_2 + P_3= 4+8+12 =24$
4^{th} Square	4 units	4 x 4 = 16 units	04	$S_4= P_1 + P_2 + P_3+P_4 = 4+8+12+16 =40$
5^{th} Square	5 units	4 x 5 = 20 units	05	$S_5= P_1 + P_2 + P_3+P_4 + P_5=4+ 8+12+16+20 =60$
……………	……….	……………	……………	……………
………………… ……	…………… ………	……………	……………	……………
n^{th} Square	n units	4 x n = 4n units		$S_n= P_1+P_2 +P_3+P_4+P_5…….P_n$ $= 4+8+12+16+20……..+ 4n =2n(n+1)$
$(n+1)^{th}$ Square		.		

Note: Blank worksheet may be provided to the students.

Conclusion:-

By using the idea of mathematical induction students can verify the result

P (n) : $4 + 8 + 12 + 16 + 20 + \ldots\ldots\ldots + 4n = 2n(n+1)$

CHAPTER 5: COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Activity 12

AIM : To visualize complex numbers, the conjugate and negative of complex numbers as points on the Argand plane (Complex Plane).

BASIC CONCEPTS:

Corresponding to every complex number z= a+ib there exists an ordered pair of real numbers (a,b) and vice versa.

MATERIAL REQUIRED:

a) Rectangular geo board of size 50cmX50cm / Graph sheets.
b) Rubber band one packet (different colours)
c) Small round stickers of different colours
d) Geometry box

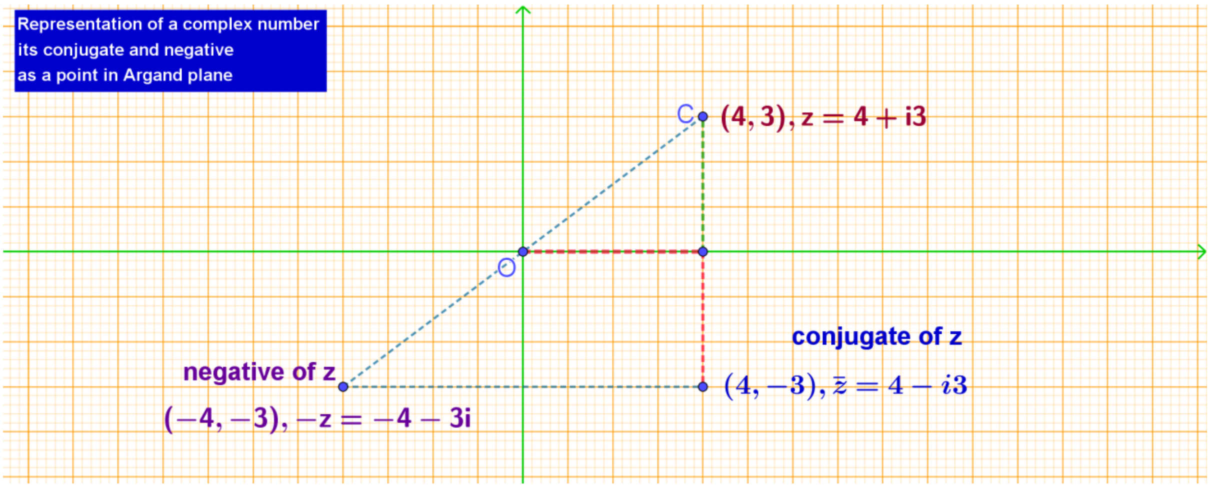
PROCEDURE:

On geo board x-axis (Real axis), y- axis (imaginary axis) are represented by colouredrubber bands. Otherwise graph sheet can also be used. Complex number z= a+ibis to be represented asan ordered pair of real numbers (a,b) Its conjugate $\bar{z} = a - ib$ is represented by (a,-b) and negative –z=-a-ib as (-a,-b) by using colored rubber bands on geoboard or by plotting on graph sheet.

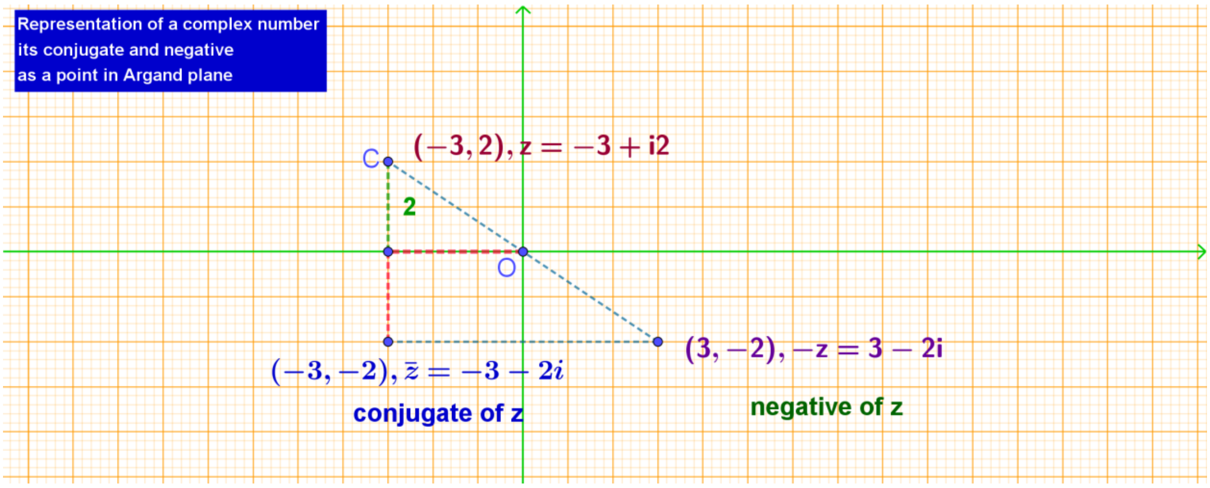
DEMONSTRATION/OBSERVATION

Complex number z=x+iy will be plotted as a point (x,y). For example the complex number z=4+3i as a point (4,3) ,its conjugate as the point (4,-3), negative of z as the point (-4,-3) are plotted on the complex plane. Similarly for the complex number z= -3+2i as shown in the adjoining figures

Complex number z=4+3i, its conjugate and negative:



Complex number z=-3+2i, its conjugate and negative:



CONCLUSION:-

Students will be able to visualize complex numbers ,its conjugate and negative graphically for richer grasping.

Activity- 13

AIM:- To find polar form r(cos θ +isin θ) of a complex number z= a+ib and to get modulus and amplitude of complex number

BASIC CONCEPTS:-

a) A complex number z= a+ib is expressed as point (a,b) in Argand plane.
b) Modulus $|z| = \sqrt{a^2+b^2}$ and amplitude of a complex number

MATERIAL REQUIRED:-

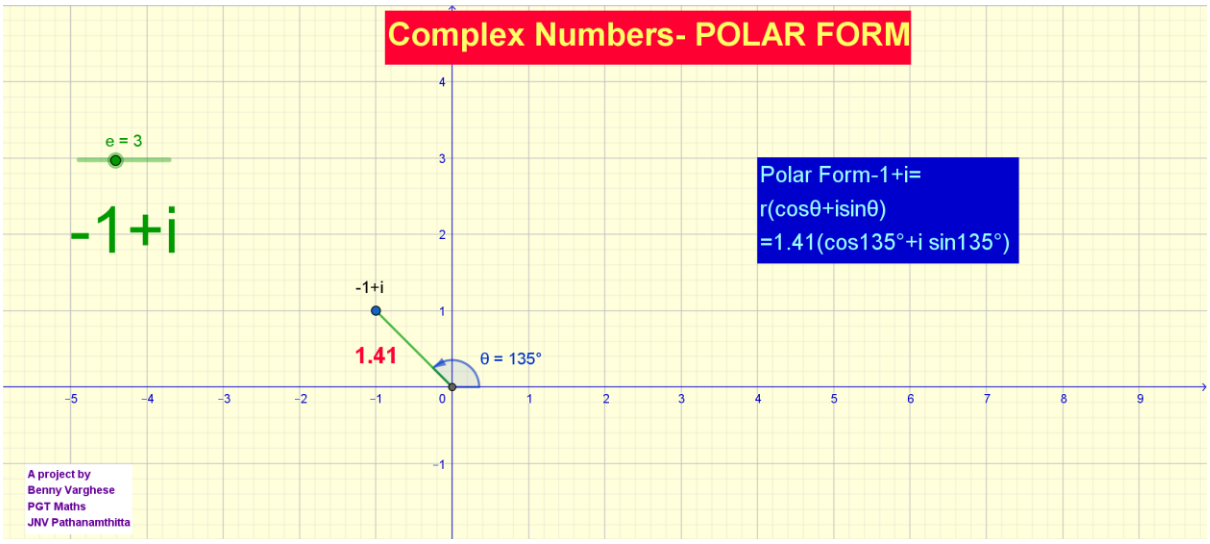
a) Graph sheet
b) Ruler
c) Geometry box
d) Protractor

PROCEDURE:-

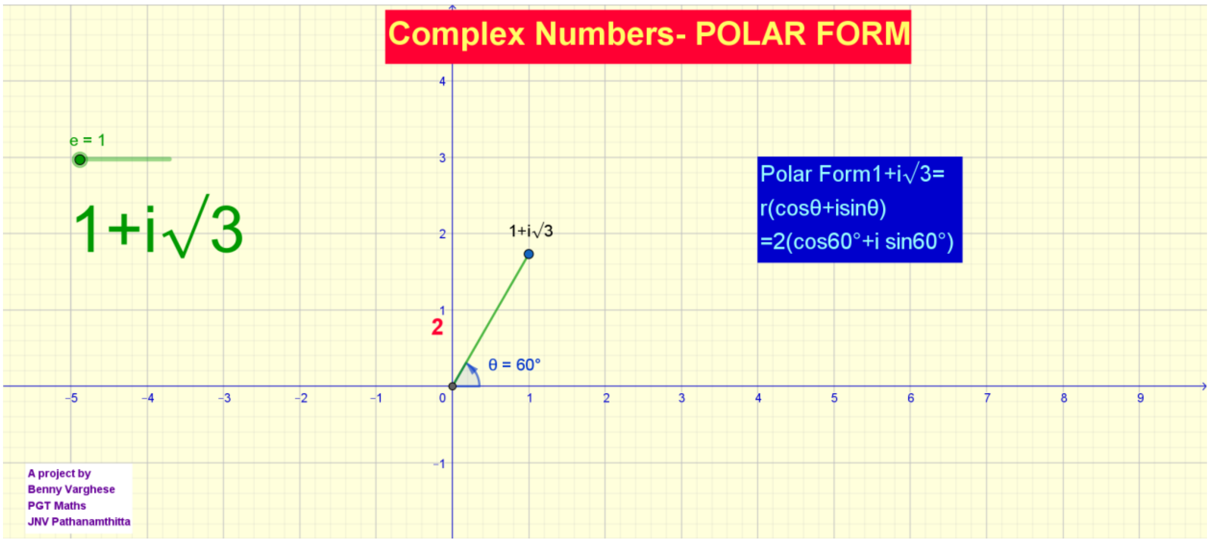
On graph sheet x-axis (Real axis), y- axes (imaginary axis) are represented and each of the given complex numbers z= a+ib is plotted as point (a,b) . Let A represent (a,b). Join OA and draw AM perpendicular to real axis, measure OA = r and $\angle$AOX = θ by using ruler and protractor.Then polar form is r(cos θ+isin θ) and polar coordinates are (r, θ)

DEMONSTRATION/OBSERVATION:-

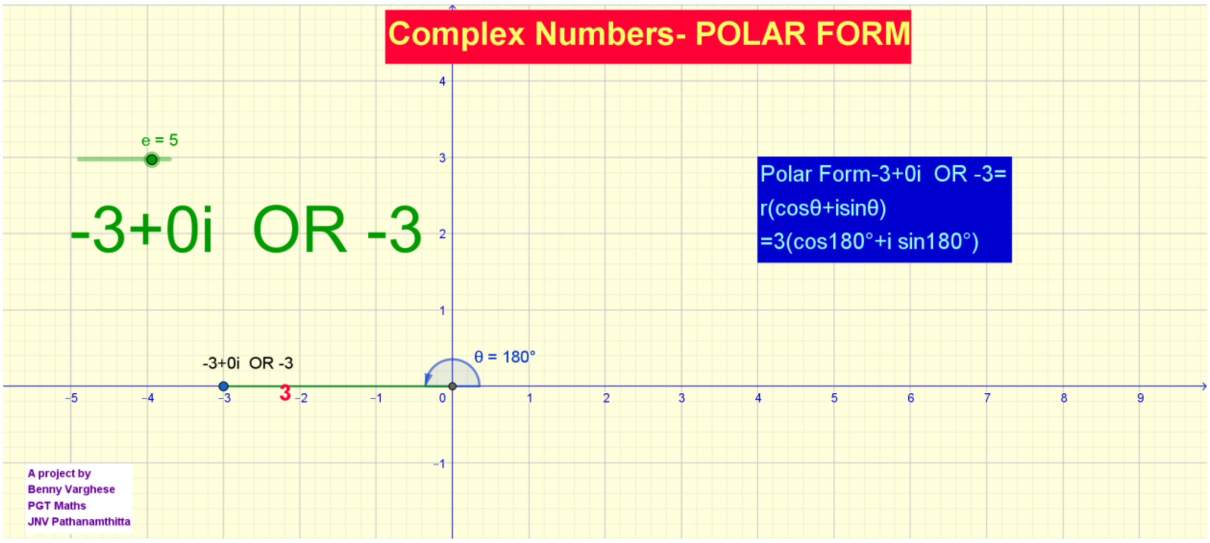
Given complex number z = -1+i will be plotted as a point(-1, 0). Measure its distance from the origin which will be the modulus of the complex number, using protractor, measure the angle made with positive direction of the x axis gives the amplitude or argument of z.



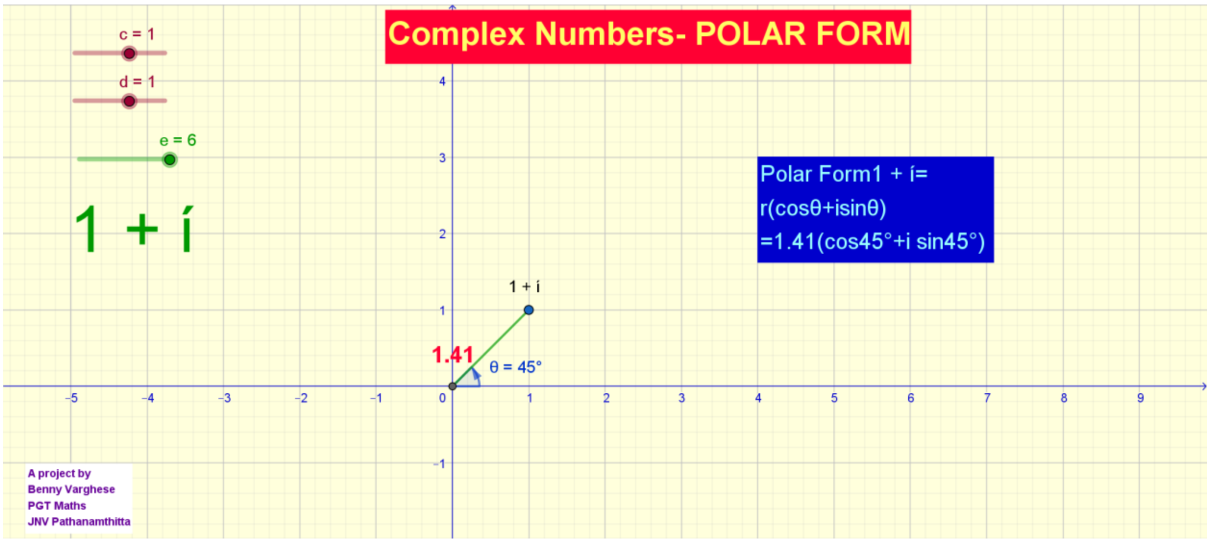
Here z=1+i√3 is to plotted as a point(1,1.73). Determine modulus and amplitude as explained above.



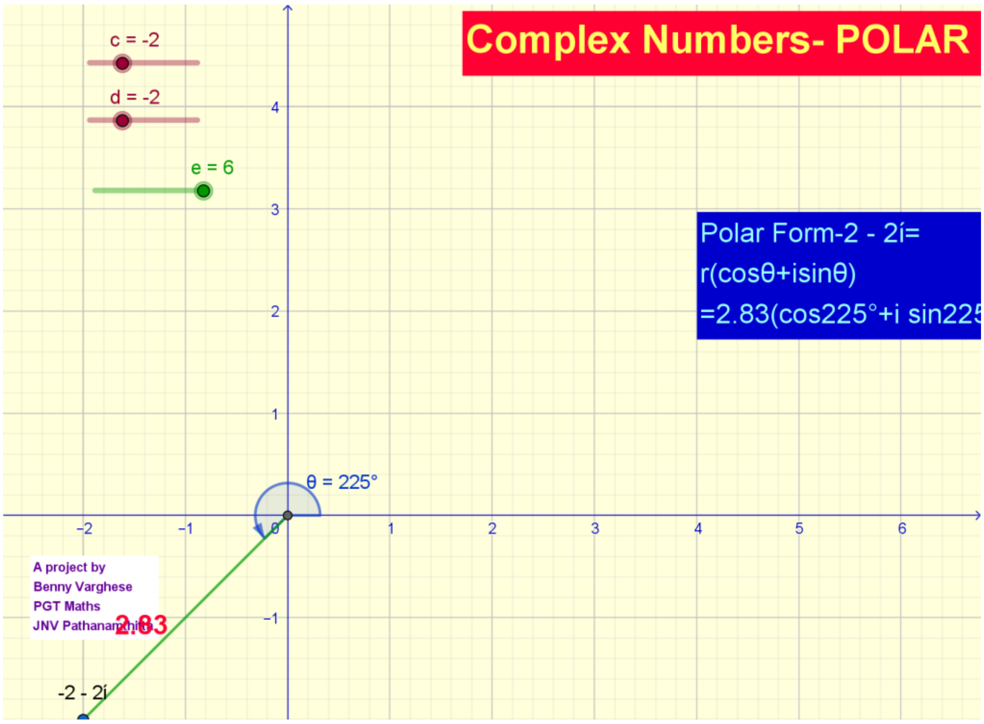
Here z=-3 that us z=-3+i0 is given. Plot the point (-3,0) and determine the modulus and amplitude of z as above.



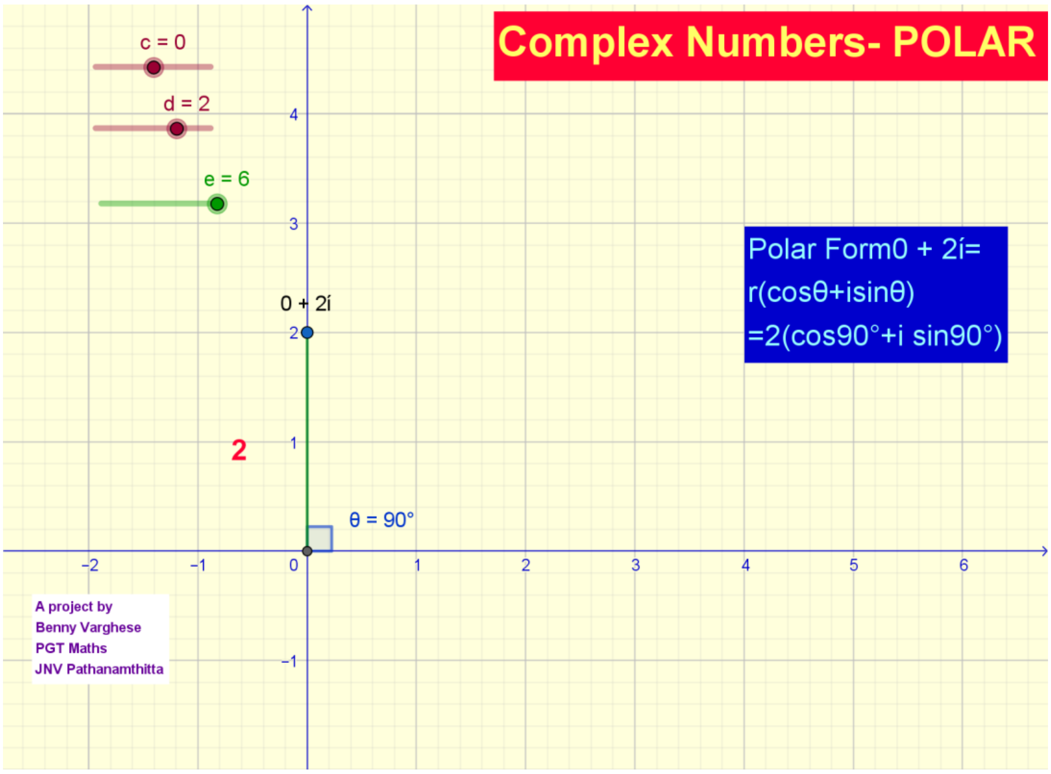
Here z=1+I is to represented as apoint(1,1) and determine the modulus and amplitude of z as above



Here z=-2-2i given. Plot the point (-2,-2) and determine the modulus and amplitude of z as above.



Here z=2i that is z = 0+2i is given. Plot the point (0,2) and determine the modulus and amplitude of z as above



Observation table:

S.No	Complex number z =a+ib	r = OA (*by* measurement)	θ = $\angle$AOX (*by* measurement)	Polar form, r(cosθ+isinθ)	Polar coordinates (r, θ)
1	Z=1+i$\sqrt{3}$	2	$\frac{\pi}{3}$	$2(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$	$(2, \frac{\pi}{3})$
2					
3					
4					
5					

CONCLUSION :-

Complex numbers are visualized in Argand plane using polar form and modulus amplitude are determined

Activity :14

AIM : To verify that for the complex numbers z_1 (r_1, α) , z_2 (r_2, β) and $Z_1 . Z_2$ = ($r_1 . r_2$, $\alpha + \beta$)

Material Required : Graph sheets and geometrical box

Pre requisite knowledge :

1. Representation of complex number into complex plane ”
2. Conversion of a complex number into its polar form
3. Multiplication of complex numbers in Cartesian form

Procedure :

- Consider the complex numbers $z_1 = 1 + i\sqrt{3}$ and $z_2 = \sqrt{3} + i$
- Find the polar form of z_1 and z_2
- You will get their polar forms$z_1 = 2(\cos 60° + i \sin 60°)$ and$z_2 = 2(\cos 30° + i \sin 30°)$
- $z_1(r_1 \ \alpha) = z_1(2, \ 60°)$ and $z_2(r_2, \ \beta) = z_2(2, 30°)$
- Find the product $z_1 . z_2 = (1 + i\sqrt{3})(\sqrt{3} \ + i)$

$$= (\sqrt{3} - \sqrt{3} + i\,4)$$

$$= 0 + 4i$$

$$= 4(0 + i)$$

$$= 4(\cos 90° + i \sin 90°)$$

$$= (4, \ 90°) \text{ polar form}$$

$$= (r, \ \theta)$$

$$where \quad r = 4 = 2 \times 2 = r_1 r_2 \ \ and \ \ \theta = 90° = 60° + 30°$$

- Represent the polar forms of z_1 , z_2 and ($z_1 . z_2$) in the argand plane
- Continue the same process with

1. $z_1 = 1 + i$ and $z_2 = 1 - i$
2. $z_1 = \frac{\sqrt{3}-1}{2} + i\frac{\sqrt{3}+1}{2}$ and $z_2 = \frac{\sqrt{3}-1}{2} - i\frac{\sqrt{3}+1}{2}$

Observations:-

		Polar forms	$z_1 . z_2$	Polar form
1	$z_1 = 1 + i\sqrt{3}$	$z_1 = (2, 60°)$	$4(0 + i)$	$z_1 z_2 = (4, 90°)$
	$z_2 = \sqrt{3} + i$	$z_2 = (2, 30°)$		
2	$z_1 = 1 + i$			
	$z_2 = 1 - i$			
3	$z_1 = \frac{\sqrt{3} - 1}{2} + i\frac{\sqrt{3} + 1}{2}$			
	$z_1 = \frac{\sqrt{3} - 1}{2} - i\frac{\sqrt{3} + 1}{2}$			

Conclusion :-

With the observations we made, it is verified that for any two complex numbers

$z_1 = (r_1, \ \alpha \,) \, and \ \ z_2 = \ (r_2, \ \beta)$, $z_1 z_2 = \ (r_1 r_2, \ \ \alpha + \beta)$

Activity :15

AIM :- To verify that for the complex numbers z_1 (r_1, α) ,

z_2 (r_2, β) and $\frac{z_1}{z_2} = \left(\frac{r_1}{r_2}, \ \alpha - \beta\right)$

Material Required :- Graph sheets and geometry box

Pre requisite knowledge :-

1. Representation of complex number into complex plane ”
2. Conversion of a complex number into its polar form
3. Multiplication and division of complex numbers in Cartesian form

Procedure :-

- Consider the complex numbers $z_1 = 1 + i\sqrt{3}$ and $z_2 = \sqrt{3} + i$
- Find the polar form of z_1 and z_2
- You will get their polar forms $z_1 = 2(\cos 60° + i \sin 60°)$ and $z_2 = 2(\cos 30° + i \sin 30°)$
- $z_1(r_1 \ \alpha) = z_1(2, \ 60°)$ and $z_2(r_2, \ \beta) = z_2(2, 30°)$
- Find the product $\frac{z_1}{z_2} = \frac{1+i\sqrt{3}}{\sqrt{3}+i}$

$$= \frac{1+i\sqrt{3}}{\sqrt{3}+i} \times \frac{\sqrt{3}-i}{\sqrt{3}-i}$$

$$= \frac{\sqrt{3}-i+3i+\sqrt{3}}{4}$$

$$= \frac{2\sqrt{3}+2i}{4}$$

$$= \frac{\sqrt{3}+i}{2}$$

$$= \frac{\sqrt{3}}{2} + i\,\frac{1}{2}$$

$$= 1(\cos 30° + i \sin 30°)$$

$$= (r, \ \theta)$$

$$Where\ r = 1 = \frac{r_1}{r_2} = \frac{2}{2}\ and$$
$$\theta = 30° = 60° - 30° = \alpha - \beta$$

- Represent the polar forms of z_1, z_2 and z_1/z_2 in the argand plane

- Continue the same process with
  1. $\boldsymbol{z_1 = 1 + i}$ **and** $\boldsymbol{z_2 = 1 - i}$
  2. $\boldsymbol{z_1 = \frac{\sqrt{3}-1}{2} + i\frac{\sqrt{3}+1}{2}}$ **and** $\boldsymbol{z_2 = \frac{\sqrt{3}-1}{2} - i\frac{\sqrt{3}+1}{2}}$

<u>Observations:-</u>

S,No	Complex numbers	Polar forms	$\boldsymbol{z_1 . z_2}$	Polar form
1	$\boldsymbol{z_1 = 1 + i\sqrt{3}}$	$\boldsymbol{z_1 = (2, 60°)}$	$\boldsymbol{\frac{z_1}{z_2} = \frac{\sqrt{3}}{2} + i\frac{1}{2}}$	$\boldsymbol{\frac{z_1}{z_2} = (1,\quad 30°)}$
	$\boldsymbol{z_2 = \sqrt{3} + i}$	$\boldsymbol{z_2 = (2, 30°)}$		
2	$\boldsymbol{z_1 = 1 + i}$	$\boldsymbol{z_1 = (\sqrt{2}, 45°)}$		
	$\boldsymbol{z_2 = 1 - i}$	$\boldsymbol{z_2 = (\sqrt{2}, -45°)}$		
3	$\boldsymbol{z_1 = \frac{\sqrt{3}-1}{2} + i\frac{\sqrt{3}+1}{2}}$	$\boldsymbol{z_1 = (\sqrt{2}, 75°)}$		
	$\boldsymbol{z_1 = \frac{\sqrt{3}-1}{2} - i\frac{\sqrt{3}+1}{2}}$	$\boldsymbol{z_2 = (\sqrt{2}, -75°)}$		

<u>Conclusion</u> :-

With the observations we made, it is verified that for any two complex numbers

$$z_1 = (r_1,\ \alpha)\ and\ \ z_2 = (r_2,\ \beta),\quad \frac{z_1}{z_2} = \left(\frac{r_1}{r_2},\quad \alpha - \beta\right)$$

ACTIVITY :16

AIM :-To verify that quadratic equation a $x^2 + bx + c = 0$ has complex solutions when $b^2 - 4ac<0$ by graphical method

Material Required :- Graph sheets and geometry box

Pre requisite knowledge :-

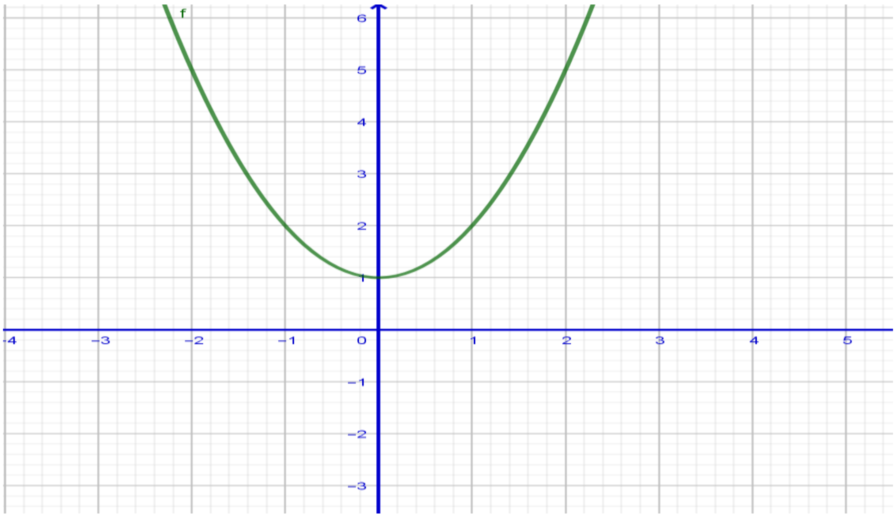
1. Fundamental theorem of algebra "A polynomial of degree n has n roots"
2. Geometrical interpretation of a quadratic polynomial as a parabolic path
3. Graphical curve of a quadratic polynomial intersects x –axis at its real solution
4. Discriminant of quadratic equation a $x^2 + bx + c = 0$ is given by $b^2 - 4ac$

Procedure :-

- Let us take the polynomial P(x) = $x^2 + 1$
- Find the value of polynomial P(x) at different values of x as given in the table

x	-4	-3	-2	-1	0	1	2	3	4
P(x)	17	10	5	2	1	2	5	10	17

- Plot the graph of polynomial by taking x values along x – axis and p(x) valuesalong y axis



- Find the points on x –axis where the curve representing polynomial $x^2 + 1$ and recall that every polynomial intersects x – axis at its real solution and a polynomial of degree n has n roots
- Compare the polynomial $x^2 + 1$ with a $x^2 + bx + c$ and get a = 1 b = 0 and c= 1 and then find the value of $b^2 - 4ac$

- Continue the same process with the equation $x^2 + x + 1 = 0$ and note your observations

Observations:-

S.No	Equation	Type of the curve	Intersecting x – axis or not	$B^2 - 4ac$	solution
1	**$x^2 + 1 = 0$**	**Open up**	**Not intersecting**	**Negative**	**Complex**
2	**$X^2 + x + 1=0$**				

1. You can observe the graph representing polynomial $x^2 + 1$ is an open up curve
2. The curve intersects y axis at (0, 1) but not intersecting x – axis at any points
3. Observe that $b^2 - 4ac< 0$

Conclusion :-

With the observations we made it is verified that the quadratic equation $ax^2 + bx + c = 0$ has no real roots when its discriminant $b^2 - 4ac < 0$ Hence solution of the above equation are available in the set of complex numbers.

CHAPTER: 6 LINEAR INEQUALITIES

Activity:-17

AIM: To represent solution set (feasible region) of a system of linear inequalities in two variables

BASIC CONCEPTS/ PREVIOUS KNOWLEDGE:

(a) To draw the graph of a linear equation in two variables

(b) To identify the half plane related to an inequality in two variables

MATERIAL REQUIRED:-

(a) Transparent Sheets
(b) Marker pen of different colours
(c) Paper clips
(d) Geometry box

PROCEDURE:

Graph of each inequality is to be drawn on separate transparent sheet. Corresponding half planes are to be shaded by using markers of different colours. By superimposing them, the common shared region will be visible. This common region represents the solution set of the given system of the inequalities

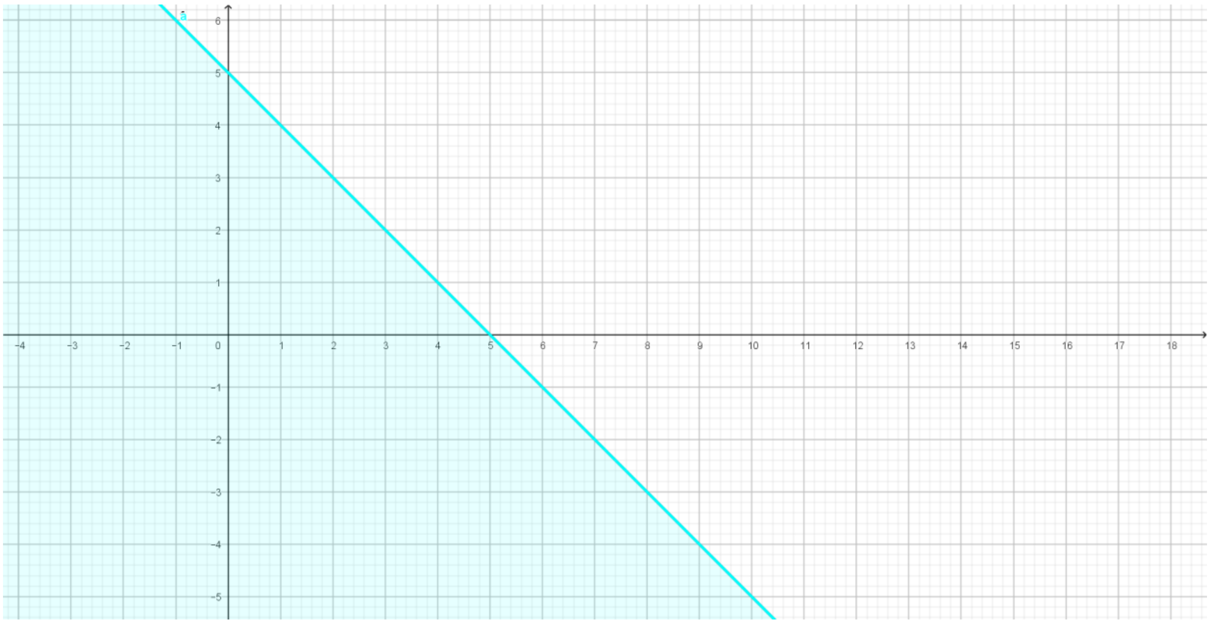
DEMONSTRATION/OBSERVATION:-

For example consider the system of linear inequalities

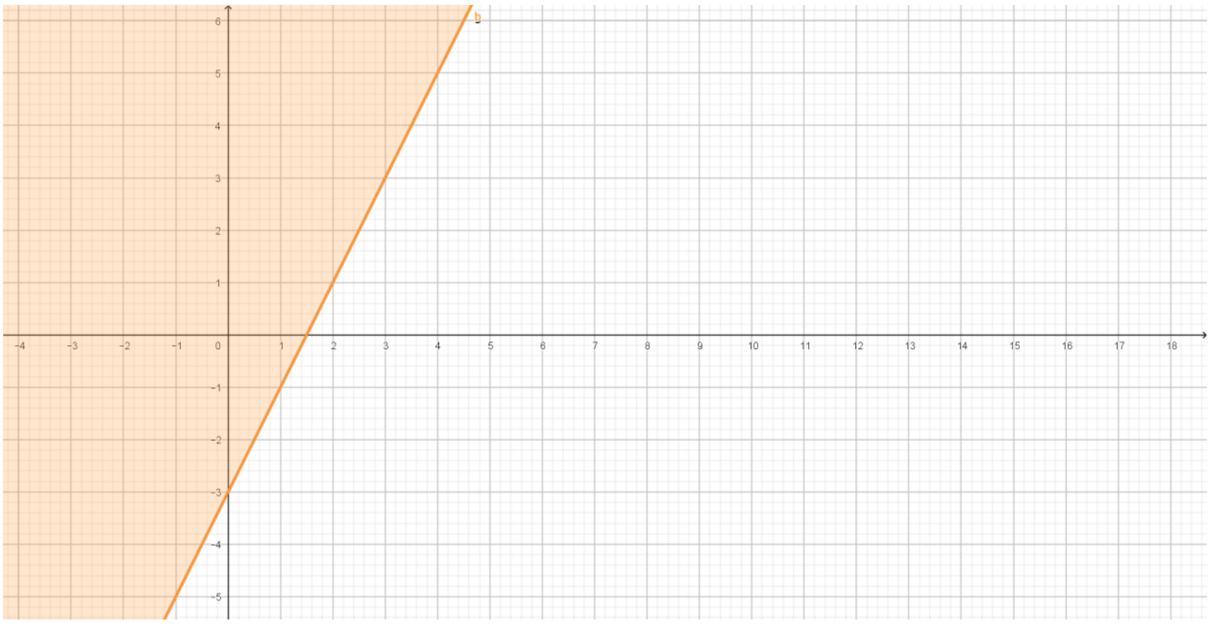
$$x+y\leq 5$$
$$2x-y\leq 3$$
$$x\geq 0$$
$$y\geq 0$$

Separate transparent sheets are used to draw the half planes corresponding to each of these inequalities as shown below:

Graph of $x+y\leq 5$



Graph of $2x-y\leq3$



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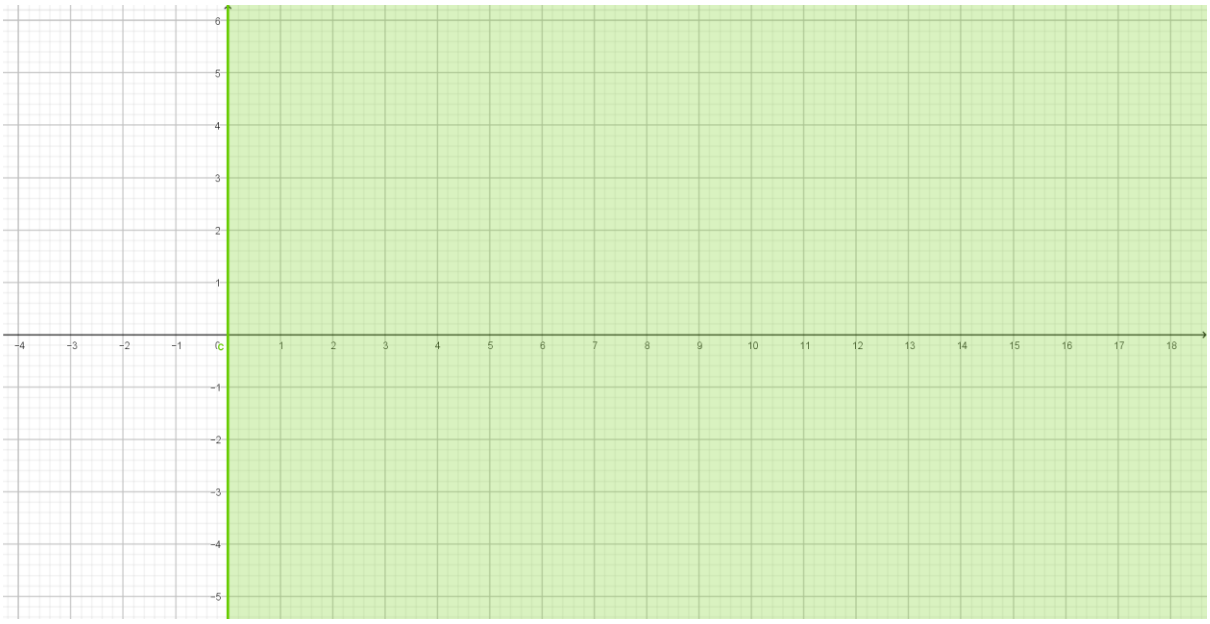
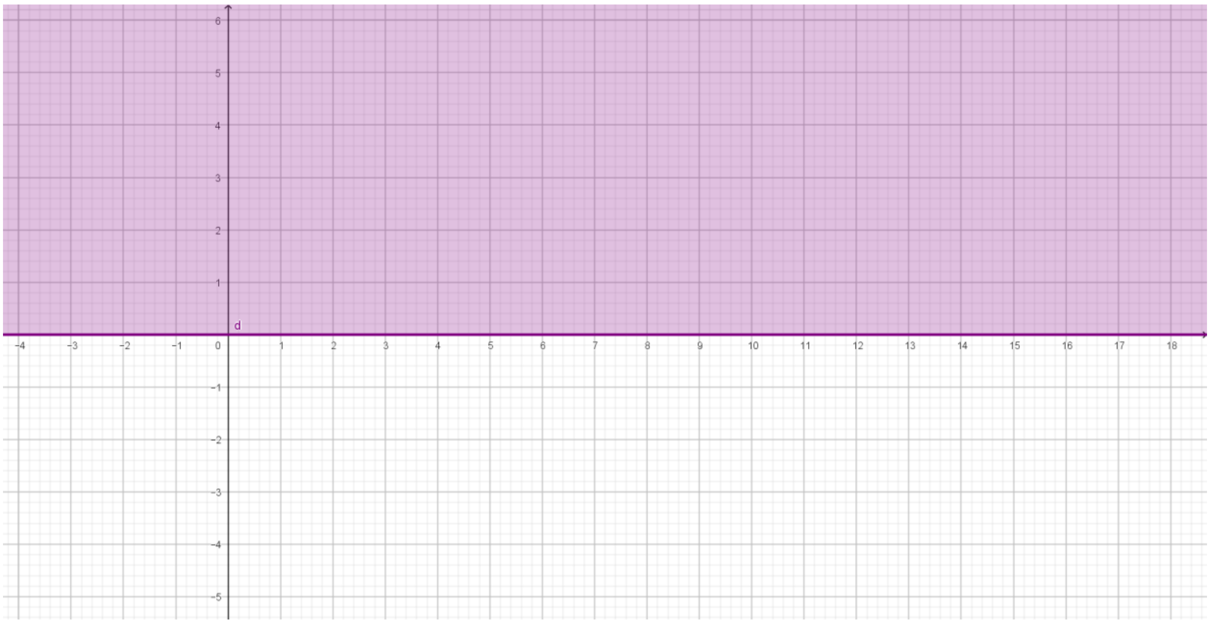
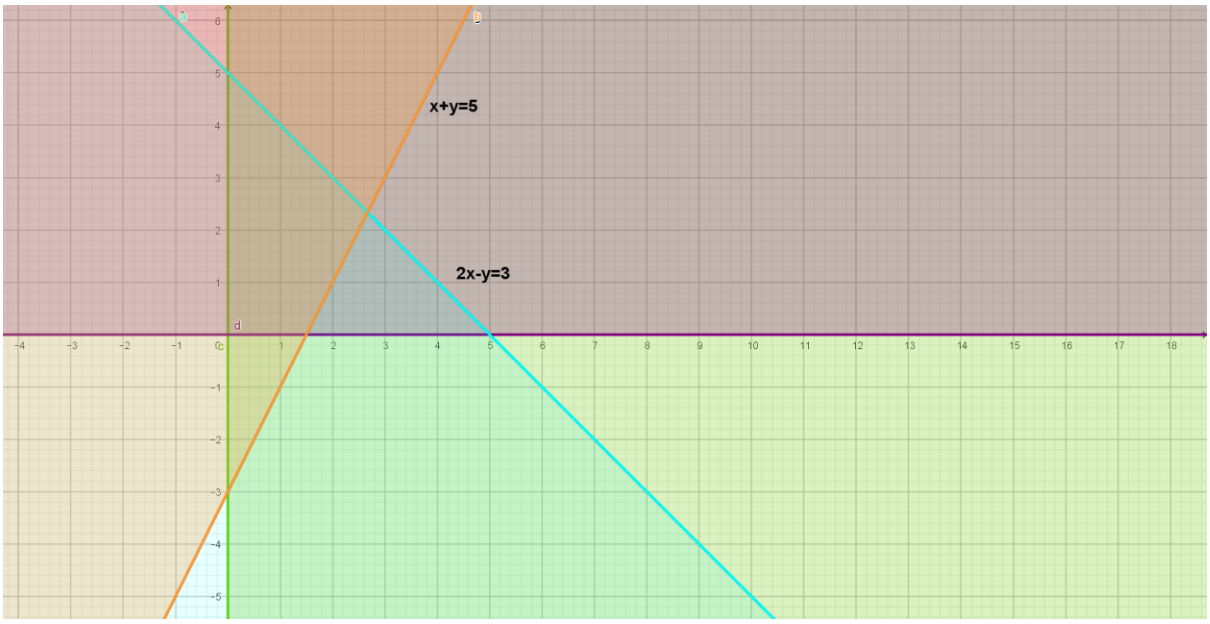
Graph of $x \geq 0$



Graph of $y \geq 0$



Superimposing all these 4 transparent sheets to get the solution set as below :



CONCLUSION :-

Solution set of a system of linear inequalities gets visualized.

Activity:18

AIM : To strengthen the concepts related to linear inequalities (as revision or post learning activity)

BASIC CONCEPTS (Pre-requisite knowledge)

a) Solving linear inequalities in one or two variables
b) To represent the solution set graphically/algebraically.

MATERIAL REQUIRED:

a) Worksheet- Print out of table having 8 X 2 columns on which 8 inequalities are printed in left columns (A4 sheets). Right columns are left blank.
b) Print out of their graphical or algebraic solutions –small chits
c) Glue stick

PROCEDURE:

Students will solve the given inequalities and identify them from the given solution chits and paste them in the column provide against each inequality.

DEMONSTRATION/OBSERVATION

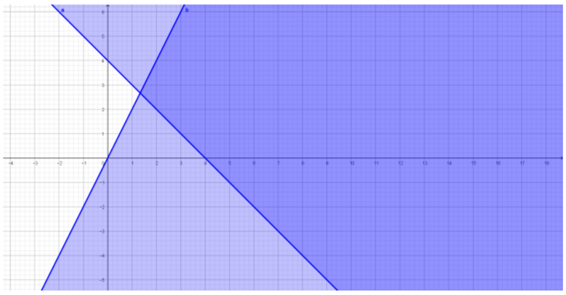
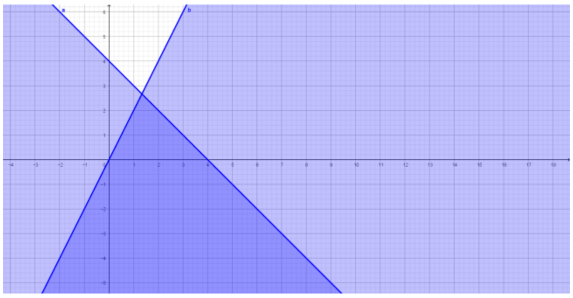
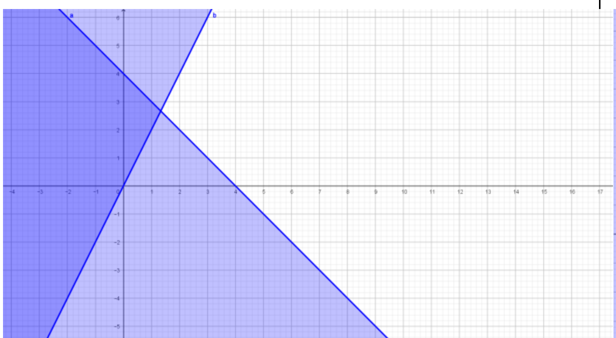
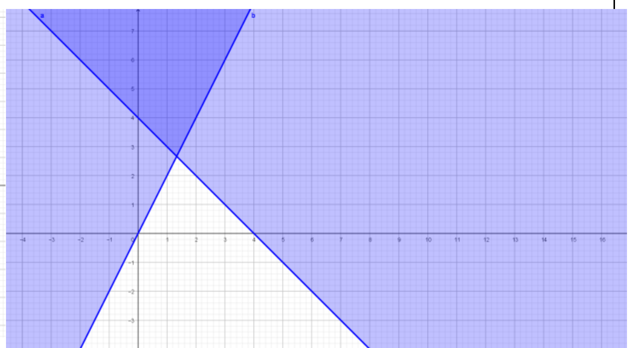
Worksheets and solution sets are printed as shown below

Work sheet

7x+3<5x+9	
$\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$	
$3(x-1) \leq 2(x-3)$	
$2 \leq 3x - 4 \leq 5$	
$x+y \geq 4, 2x-y \geq 0$	

$x+y\leq 4, 2x-y\geq 0$	
$x+y\leq 4, x+y\geq 0$	
$x+y\leq 4, x+y\leq 0$	

<u>Solution Sets;-</u> (which are to be identified and to be pasted in worksheet against each inequality**)**

[2,3]	**X$\leq$-3**
x$\geq$1	**X<3**

CONCLUSION:

To promote skills of solving linear inequalities and to identify their solution sets.

CHAPTER:-7 PERMUTATIONS AND COMBINATIONS
Activity-19

AIM: To prove theory of fundamental principle of counting.

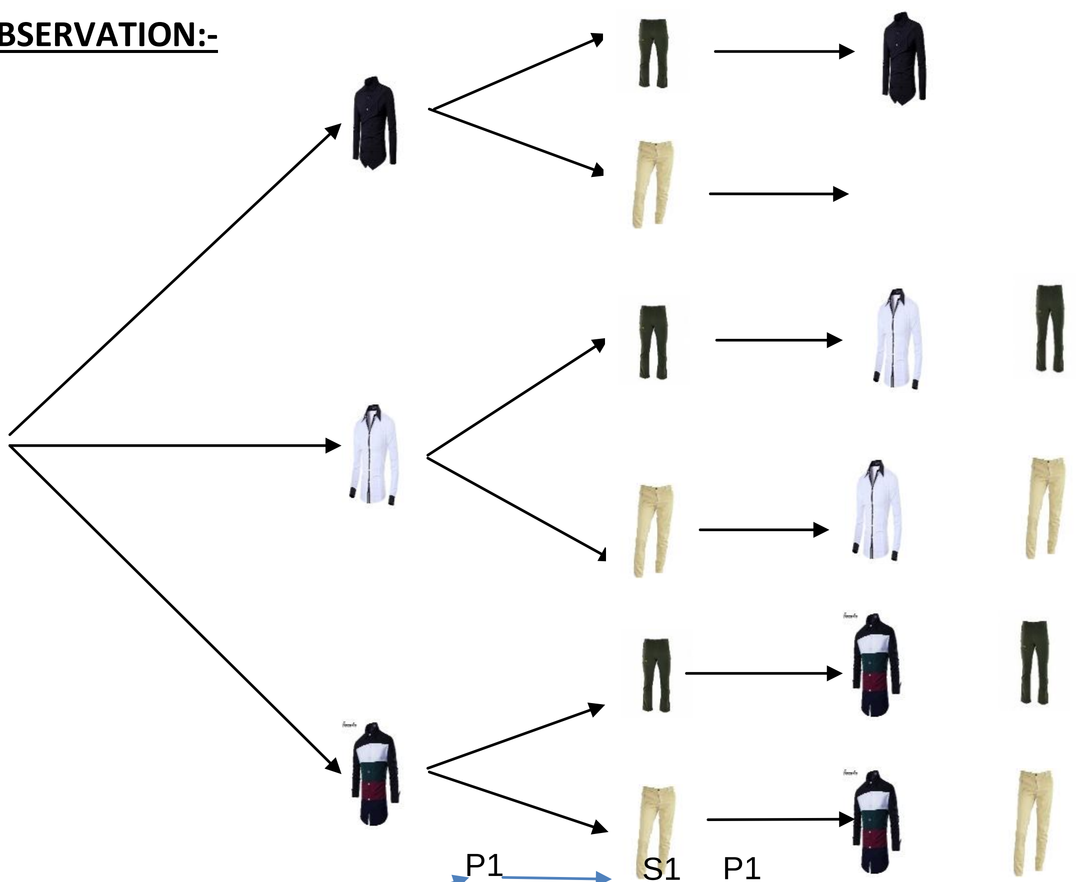
MATERIALS REQUIRED: Card Board, White chart papers, Colored pens, Pencil, Eraser, Adhesive and scissors.

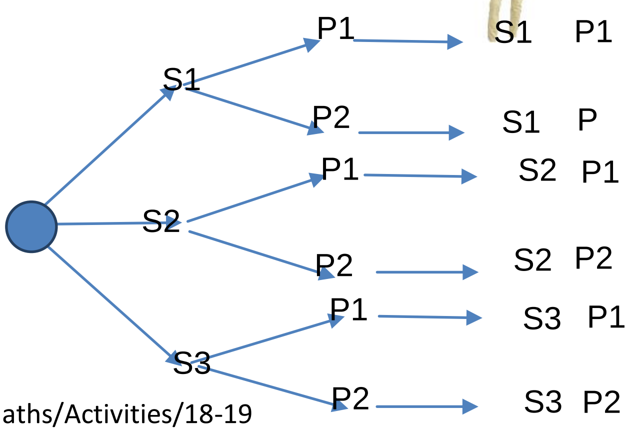
PROCEDURE: CASE - I

1. Take a card board of convenient size and paste a white chart paper on it.
2. Make two pants and three shirts of various colors by cutting white chart paper and colored them.
3. Indicate two pants as P_1 and P_2 similarly three shirts as S_1, S_2 and S_3.

S_1 S_2 S_3 P_1 P_2

OBSERVATION:-





OBSERVATION TABLE:-

Shirts	Pants	Pair of shirt & pant	Total no. of pairs
S_1	P_1	$S_1 P_1$ $S_1 P_2$	Shirts→ 3 & Pants→ 2 = 3 x 2 = 6
S_2	P_2	$S_2 P_1$ $S_2 P_2$	
S_3		$S_3 P_1$ $S_3 P_2$	

CASE – II

1. Take a card board of convenient size and paste a white chart paper on it.
2. Make five water bottles and four lunch boxes of various cutting white chart paper and colored them. 

3. Indicate five water bottles as $(B)_1$, $(B)_2$, $(B)_3$, $(B)_4$ and $(B)_5$ similarly four lunch boxes as $(L)_1$, $(L)_2$, $(L)_3$and $(L)_4$.

OBSERVATION:-

Water Bottles	Lunch Boxes	Pair of Water Bottles Lunch Boxes	Total no. of pairs
$(B)_1$	$(L)_1$	$(B)_1(L)_1$, $(B)_1(L)_2$ $(B)_1(L)_3$, $(B)_1(L)_4$	Water Bottles→ 5
$(B)_2$	$(L)_2$	$(B)_2(L)_1$, $(B)_2(L)_2$ $(B)_2$,$(L)_3$,$(B)_2(L)_4$	Lunch Boxes→ 4
$(B)_3$		$(B)_3(L)_1$, $(B)_3(L)_2$ $(B)_3$,$(L)_3$,$(B)_3(L)_4$	Total Number of Arrangements
$(B)_4$	$(L)_3$	$(B)_4(L)_1$, $(B)_4(L)_2$ $(B)_4(L)_3$, $(B)_4(L)_4$	= 5 x 4
$(B)_5$	$(L)_4$	$(B)_5(L)_1$, $(B)_5(L)_2$ $(B)_5(L)_3$,$(B)_5(L)_4$	= 20

CONCLUSION:-

If an event occur in **'m'** different ways and another event occur in 'n' different ways, then the total number of occurrence of the events in the given order is **m x n** ways.

Activity – 20

AIM:- Derive formula for permutation when all the objects are distinct
(if repetition is allowed).

MATERIALS REQUIRED:- Card board, White chart paper, colored pens, thumb pin, scissor, scale etc.

PROCEDURE:-

REPETITION IS ALLOWED:-

1. We take seven different digits (1, 2, 3, 4, 5, 6 and 7)
2. First place can be filled in 7 different ways by any one of the 7 digits.

7ways
I - place

3. Total number of single digit number = 7 = 7^1
4. After the filled in first place we have again 7 digits are left.
5. Second place can be filled in again 7 different ways by any one of the 7 digits.

7ways	7ways	=7 x 7 = 49 = 7^2
I - place	II - place	

6. Total number of 2 digits number = 49
7. After the filled in second place we have again 7 digits are left.
8. Third place can be filled in again 7 different ways by any one of the 7 digits.

7ways	7way	7ways	= 7 x 7 x 7 = 343 = 7^3
I - place	II – place	place -III	

9. Total number of 3 digits number = 343
10. After the filled in third place we have again 7 digits are left.
11. Fourth place can be filled in again 7 different ways by any one of the 7 digits.

7ways	7ways	7ways	7ways	= 7 x 7 x 7 x 7 = 2401 = 7^4
I - place	II – place	III – place	IV - place	

Similarly

12. Total number of 4 digits numbers = 7x7x7x7 = 2401 = 7^4
13. Total number of 5 digits numbers = 7x7x7x7x7 = 16807 = 7^5
14. Total number of 6 digits numbers = 7x7x7x7x7x7 = 117649 = 7^6
15. Total number of 7 digits numbers = 7x7x7x7x7x7x7 = 823543 = 7^7
16. Hence we take n different objects and taken r at a time, then total number of permutations = n^r (if repetition is allowed)

OBSERVATION TABLE:-

Sl. No.	Total No. of Diff. Objects (n)	Taken objectsat a time (r)	Total No. of arrangements	Formula in Permutations
1	7	1	$7 = 7 = 7^1$	Total number of permutations of n diff. objects taken r at a time (if repetition is allowed) = $\mathbf{n^r}$
2	7	2	$7 \times 7 = 49 = 7^2$	
3	7	3	$7 \times 7 \times 7 = 343 = 7^3$	
4	7	4	$7 \times 7 \times 7 \times 7 = 2401 = 7^4$	
5	7	5	$7 \times 7 \times 7 \times 7 \times 7 = 16807 = 7^5$	
6	7	6	$7 \times 7 \times 7 \times 7 \times 7 \times 7 = 117649 = 7^6$	
7	7	7	$7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 823543 = 7^7$	

CONCLUSION:-

Total number of permutations of **n** different objects taken **r** at a time, if repetition is allowed is $\mathbf{n^r}$.

Activity – 21

AIM:- Derive formulae for permutation when all the objects are distinct (if repetition is not allowed).

MATERIALS REQUIRED:- Card board, White chart paper, colored pens, thumb pin, scissors, scale, etc.

PROCEDURE:

(REPETITION IS NOT ALLOWED)

01. We take seven different digits (1, 2, 3, 4, 5, 6 and 7)
02. First place can be filled in 7 different ways by any one of the 7 digits.

7ways
I - place

03. Total number of single digit number = 7 = ${}^{7}P_{1}$
04. After the filled in first place we have again 6 digits are left.
05. Second place can be filled in 6 different ways by any one of the 6 digits.

7ways	6ways	= 7 x 6 = 42
I - place	II - place	

06. Total number of two digits numbers = 42 = ${}^{7}P_{2}$
07. After the filled in second place we have again 5 digits are left.
08. Third place can be filled in 5 different ways by any one of the 5 digits.

7ways	6ways	= 7 x 6 x	5ways
I - place	II - place		III - place

09. Total number of 3 digits number = 210 = ${}^{7}P_{3}$
10. After the filled in third place we have again 4 digits are left.
11. Fourth place can be filled in 4 different ways by any one of the 4 digits.

7ways	6ways	5ways	4ways	= 7 x 6 x 5 x 4
I - place	II – place	III – place	IV - place	

Similarly

12. Total number of 4 digits numbers = 7x6x5x4 = 840 = ${}^{7}P_{4}$
13. Total number of 5 digits numbers = 7x6x5x4x3 = 2520 = ${}^{7}P_{5}$
14. Total number of 6 digits numbers = 7x6x5x4x3x2 = 5040 = ${}^{7}P_{6}$
15. Total number of 7 digits numbers = 7x6x5x4x3x2x1 = 5040 = ${}^{7}P_{7}$

16. Hence we take **n** different objects and taken **r** at a time, then total number of permutations = $\mathbf{^{n}P_{r}}$ (if repetition is not allowed)

OBSERVATION TABLE:-

Sl. No.	Total No. of Diff. Objects (n)	Taken objectsat a time (r)	Total No. of arrangements	Formula in Permutations
1	7	1	7 = 7 = $^{7}P_{1}$	Total number of permutations of n diff. objects taken r at a time (if repetition is not allowed) = $^{n}P_{r}$
2	7	2	7x6 = 49 = $^{7}P_{2}$	
3	7	3	7x6x5 = 343 = $^{7}P_{3}$	
4	7	4	7x6x5x4 = 2401 = $^{7}P_{4}$	
5	7	5	7x6x5x4x3 = 16807 = $^{7}P_{5}$	
6	7	6	7x6x5x4x3x2 = 117649 = $^{7}P_{6}$	
7	7	7	7x6x5x4x3x2x1 = 823543 = $^{7}P_{7}$	

CONCLUSION:-

Total number of permutations of n diff. objects taken r at a time, if repetition is allowed is $^{n}P_{r}$.

<u>Activity :– 22</u>

<u>AIM:</u> - To find the number of combinations.

MATERIALS REQUIRED: - Card board, White chart paper, colored pens, thumb pin, scissor, scale etc.

PROCEDURE:-

01. Call four student's (Mohan, Sohan, Raju and Pappu) of the class.
02. These four student's stand in the line.
03. Instruct these student's, hand shake with each other.
04. First call Mohan and say to hand shake with other three students.
05. Total number of hand shake by Mohan is three.

Mohan → Sohan → Mohan, Sohan
Mohan → Raju → Mohan, Raju
Mohan → Pappu → Mohan, Pappu

06. Next call Sohan, Sohan already hand shake with Mohan, then the total number of hand shake by Sohan is two.

Sohan → Raju → Sohan, Raju
Sohan → Pappu → Sohan, Pappu

07. Next call Raju, Raju already hand shaked with Mohan and Sohan then the total number of hand shake by Raju is one.

Raju → Pappu → Raju, Pappu

08. All four students hand shaked with each other.
09. Total number of hand shaked with each by four students = 6.

OBSERVATION TABLE - 1:-

Name of Student	Hand shake with each other	Total No. of Hand shake
Mohan	(Mohan, Sohan),(Mohan, Raju),(Mohan, Pappu)	3
Sohan	(Sohan, Raju),(Sohan, Pappu) Sohan already hand shaked with Mohan	2
Raju	(Raju, Pappu) Raju already hand shaked with Mohan and Sohan	1
Pappu	Pappu already hand shaked with Mohan, Sohan and Raju	0
Total No. of hand shake with each other		6

10. Similarly this can apply for ‘n’ students.

OBSERVATION TABLE – 2:-

No. of Student (n)	No. of hand shake with each other	Students required for hand shake (r)	Find ${}^nC_r = \frac{n!}{(n-r)!r!}$
n = 5	4+3+2+1 = 10	r =2	$\frac{5!}{(5-2)!2!}$ = 10
n = 6	5+4+3+2+1 = 15	r =2	$\frac{6!}{(6-2)!2!}$ = 15
n =7	6+5+4+3+2+1 = 21	r =2	$\frac{7!}{(7-2)!2!}$ = 21
n = 8	7+6+5+4+3+2+1 = 28	r =2	$\frac{8!}{(8-2)!2!}$ = 28
.	.	.	.
.	.	.	.
.	.	.	.
n = r	(r – 1) + (r – 2) +…	r =2	$\frac{n!}{(n-r)!r!} = {}^nC_r$

11. Total number of combination of ‘n’ students = $\frac{n!}{(n-r)!r!} = {}^nC_r$

Conclusion: Formula for finding the number of combinations of n different objects taken r at a time, denoted by = $\frac{n!}{(n-r)!r!} = {}^nC_r$. 

Activity – 23

AIM:- To find the formula for permutation of n objects, where p objects are of same kind and rest are all different.

MATERIALS REQUIRED:- Card board, White chart paper, colored pens, thumb pin, scissor, scale etc

PROCEDURE:-

1. We take the word **RAMA.**
2. These words are written as in a dictionary.
3. Order of letters of RAMA in English alphabet,

First $\longrightarrow$ A, A

Second $\longrightarrow$ M

Third $\longrightarrow$ R

4. Take first letter A and fixed at first place, then we can written another three letters A, M and R in English alphabet.

A	A	M	R
A	A	R	M
A	M	A	R
A	M	R	A
A	R	A	M
A	R	M	A

5. Take second letterM and fixed at first place, then we can written another three letters A, A and R in English alphabet.

M	A	A	R
M	A	R	A
M	R	A	A

6. Take third letter R and fixed at first place, then we can written another three letters A, A and M in English alphabet.

R	A	A	M
R	A	M	A
R	M	A	A

OBSERVATION TABLE - 1:-

Letter fixed at first place	Total number of arrangements	Total
A	AAMR,AARM,AMAR,AMRA,ARAM,ARMA	6
M	MAAR,MARA,MRAA	3
R	RAAM,RAMA,RMAA	3
Total No. of arrangements for RAMA with and without meaning in a dictionary.		12

7. In the word RAMA total number of objects n = 4 and same kind objects p = 2 then $\frac{n!}{p!} = \frac{4!}{2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1} = 12$
8. Similarly in the word RAMAN

OBSERVATION TABLE - 2:-

Letter fixed at first place	Total number of arrangements	Total
A	AAMNR, AAMRN, AANMR, AANRM, AARMN, AARNM, AMANR, AMARN, AMNAR, AMNRA, AMRAN, AMRNA, ANAMR, ANARM, ANMAR, ANMRA, ANRAM, ANRMA, ARAMN, ARANM, ARMAN, ARMNA, ARNAM,ARNMA	24
M	MAANR, MAARN, MANAR, MANRA, MARAN, MARNA, MNAAR, MNARA, MNRAA, MRAAN, MRANA, MRNAA	12
N	NAAMR, NAARM, NAMAR, NAMRA, MARAM, NARMA, NMAAR, NMARA, NMRAA, NRAAM, NRAMA, NRMAA	12
R	RAAMN, RAANM, RAMAN, RAMNA, RANAM, RANMA, RMAAN, RMANA, RMNAA, RNAAM, RNAMA, RNMAA	12
Total No. of arrangements RAMAN with and without meaning in a dictionary.		60

1. In the word RAMA total number of objects n = 4 and same kind objects p = 2 then $\frac{n!}{p!} = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 60$

Conclusion:-

The number of permutations of n objects, where p objects are of the same kind and rest are all different = $\frac{n!}{p!}$

CHAPTER :-8 BINOMIAL THEOREM

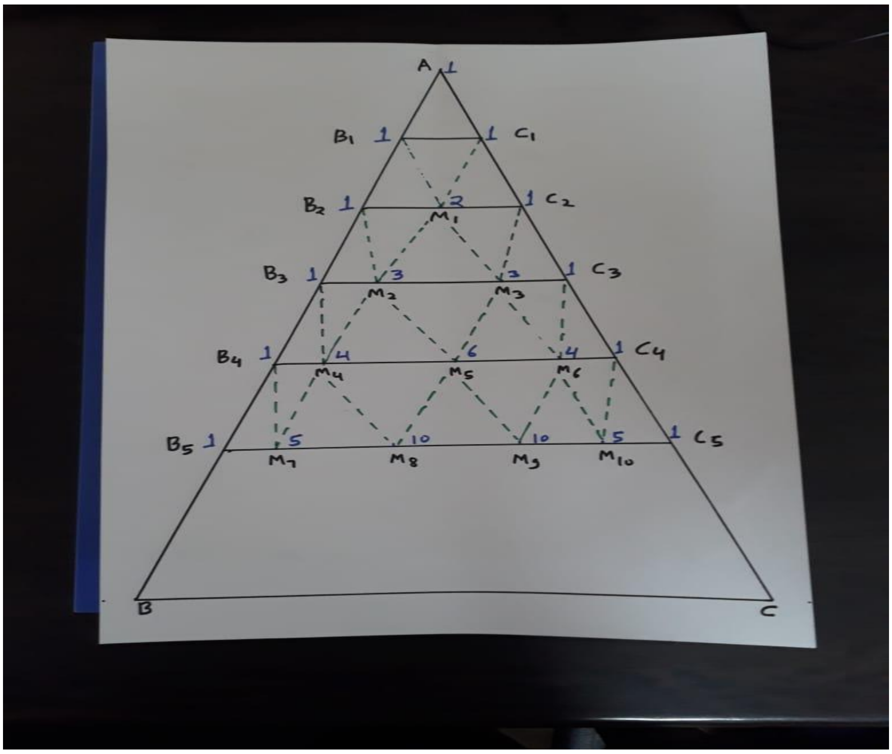
Activity-24

AIM: To find the coefficients of Binomial expansion.

MATERIALS REQUIRED: Card Board, White chart Papers, Colored Pens, Pencil, Eraser, scissor and Thumb Pin.

PROCEDURE:-

1. Draw an equilateral triangle ABC on white paper and also draw five lines parallel to base BC of triangleABC that are B_1C_1, B_2C_2, B_3C_3,B_4C_4,B_5C_5 .Write 1 one each point A_1,B_1,C_1,B_2,C_2,B_3,C_3,B_4,C_4 , B_5 , C_5 ,B_6 , C_6.
2. Write the Sum of B_1&C_1 number i.e. 1+1=2 say(M_1)
3. Write the Sum of $B_2 + M_1 = 1+2 = 3$ onM_2
4. Write the Sum of $C_2 + M_1 = 3$ onM_3
5. Write the Sum of $B_3 + M_2 = 1+3 = 4$ onM_4, ,$M_2 + M_3 = 3+3 = 6$onM_5, $M_3 + C_3 = 1+3 = 4$on$M_6$and so on……….



OBSERVATION TABLE:-

INDEX	Coefficients
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1

Result: - Similarly write the coefficient for index 6 ?

Conclusion: - By symmetry of triangle student's can easily find out coefficient of Binomial expression.

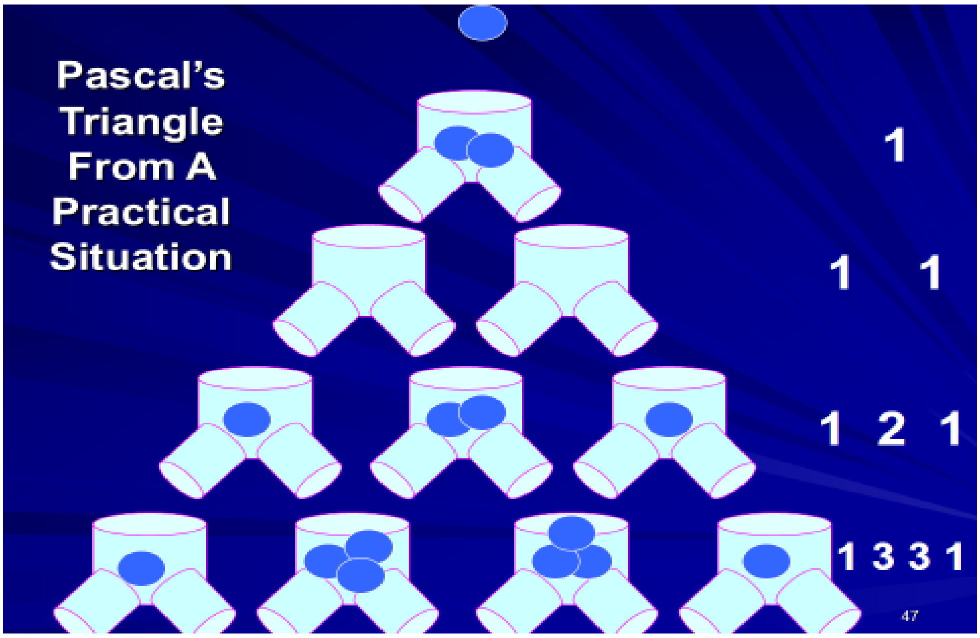
Activity -25

AIM:- To find the coefficient's of Binomial expansion.

MATERIALS REQUIRED:- 21 plastic keep of two mouth, 42 marbles, card board, 21screw, one white chart paper.

PROCEDURE:- Paste white chart paper on card board.

1. Fix plastic keeps on one chart paper by screw in an equilateral triangle way as in first row 1, second row 2, third row 3, fourth row 4 fifth row 5 plastic keeps with cover holes by small card circular boards.
2. Took two marbles and put them in 1^{st} row, keep and drop one in LHS hole & one in RHS hole (takeout card board cover from first row keeps).
3. Took two marble's and put them in 2^{nd} row of first keep & drop one marble in LHS hole & one marble in RHS hole (takeout card board cover from second row keep). Similarly took two marbles and put them in second keep of second row & droop one in LHS hole & one in RHS hole, So, We get 1 , 1+1 ,1 marbles in second row .
4. Took twice number of second marbles & put them half in LHS &half in RHS whole of keep. So, we get 1,3,3,1 marbles in third row.
5. Similarly by this method we get 1,4,6,4,1 marbles in fourth row……….



Result: - Similarly write the coefficient of index 5 ?

Conclusion: - By Pascal's triangle symmetry student Can easily, find out coefficient of Binomial expression.

CHAPTER 9 : SEQUENCE AND SERIES

Activity : 26

AIM:-To find the sum of first n terms of an arithmetic progression (A.P.).

Pre-requisite knowledge:-

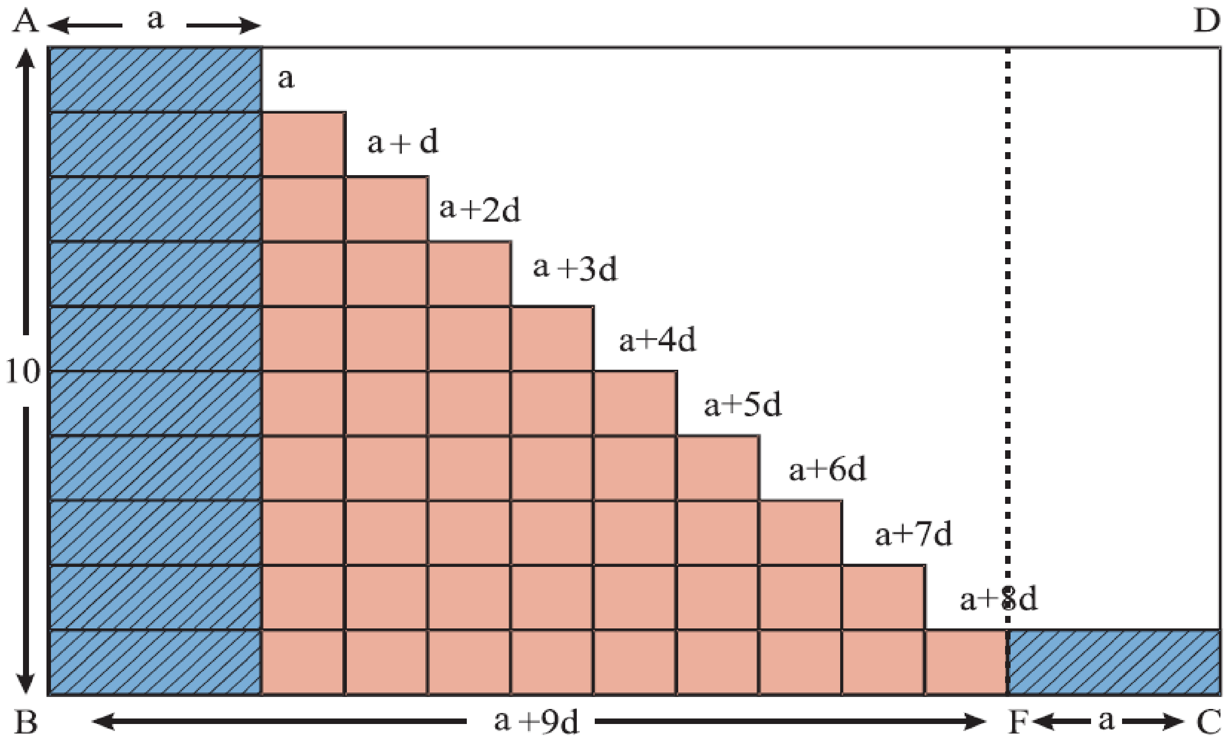
: Knowledge of arithmetic progression.

Materials required:-

(i) Plastic strips
(ii) Coloured chart paper
(iii) Thermocol sheets
(iv) Fevicol
(v) Pair of scissors
(vi) Scale, pencil and eraser

Preparation for the Activity :-

(i) Take a rectangular thermocol sheet ABCD.
(ii) Cut some plastic strips of equal fixed length, denoted by a and some others of equal
length denoted by d.
(iii) Arrange and paste both types of strips so as to get terms, a, a+d, a+2d, ……. a+9 d
placed at unit distance apart and arrange along the rectangle, as shown above
(iv) The last strip ends in F on BC, extend F To C by a fixed length a.



Demonstration and Use:-

(i) The first strip is of length a
(ii) Second strip is of length a+d
(iii) Third strip is of length a+2d
(iv) Tenth strip is of length a + 9d.
(v) Strips arranged look like a stair case
(vi) The sum of above arithmetic progression
= a + (a+d) + (a+2d) +……………+ (a+9d)
= 10a + 45d
= 5 (2a+9d) =1/2 .10. (2a +9d) = 1/2. 10 [2a +(10-1)d]
= 1/2 (Area of rectangle ABCD, where length BC =2a +9 and breadth is 10 units)

Conclusion:-

If the arithmetic progression is a, a+d, a+2d, ……..a+ (n-1)d, then the sum of its first n terms

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

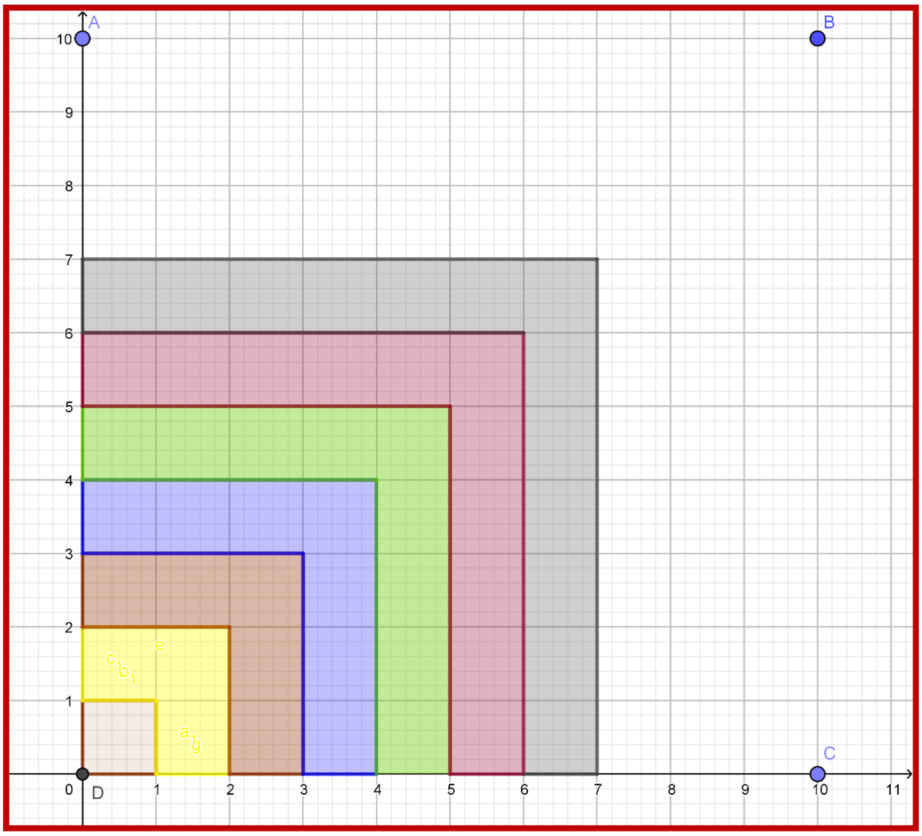
Activity : 27

AIM:-To find the Sum of first n odd natural numbers.

Pre-requisite knowledge:-
(i) Odd natural numbers
(ii) nth odd natural number can be written as 2n-1.

Materials required:-
(i) White chart paper
(ii) Scale, pencil and eraser
(iii) Coloured ball point pens
(iv) Pair of scissors
(v) Geometrical instruments

Preparation for the activity:-
(i) Take a white chart paper and cut out a square of size 10cm x 10 cm from it and mark
the boundary of the square.



(ii) Draw horizontal and vertical lines in the square to make small squares of size
1 cm x 1 cm as shown in the figure given below. (Fisg.(i))
(iii) Colour the small squares with different colours with the help of coloured pens as shown
in the figure.

Demonstration and Use:-

Number of brown coloured small squares is one.
Number of yellow coloured small squares are three.
Number of dark brown coloured small squares are five.
Number of blue coloured small squares are seven.
Number of green blue coloured small squares are nine.
Number of red coloured small squares are eleven.
Number of black coloured small squares are thirteen.

Now total number of small squares (brown =1) in 1 cm x 1cm square is1 $=1^2$
Total number of small squares (brown + yellow = (1+3) in 2cm x 2cm square is 4 = 2^2
Total number of small squares (brown + yellow+ dark brown = (1+3+5) in 3cm x3cm square is 9 $=3^2$
Total number of small squares (brown + yellow + dark brown+ blue = (1+3+5+7) in 4cm x4cm
square is 16 $=4^2$
Total number of small squares (brown + yellow + dark brown + blue + green = (1+3+5+7+9) in 5cm x5cm square is 25 $=5^2$
Total number of small squares (brown + yellow + dark brown + blue + green+ red = 1+3+5+7+9+11) in 6cm x6cm square is 36 $=6^2$
Total number of small squares(brown + yellow + dark brown + blue + green + red+ black
= 1+3+5+7+9+11+13) in 6cm x6cm square is 49 $=7^2$

.
.
.
.

and so on ....

Conclusion:-

Proceeding in thus way we observe that the total number of small squares in an n cm x n cm
square is 1+3+5+7+9+11+ ………+ (2n-1) = n^2
Hence we can say that the sum of first n odd natural numbers is n^2.

Activity : 28

AIM:- To express a natural number (say x) as a sum of infinite numbers of a sequence.

Pre-requisite knowledge :-

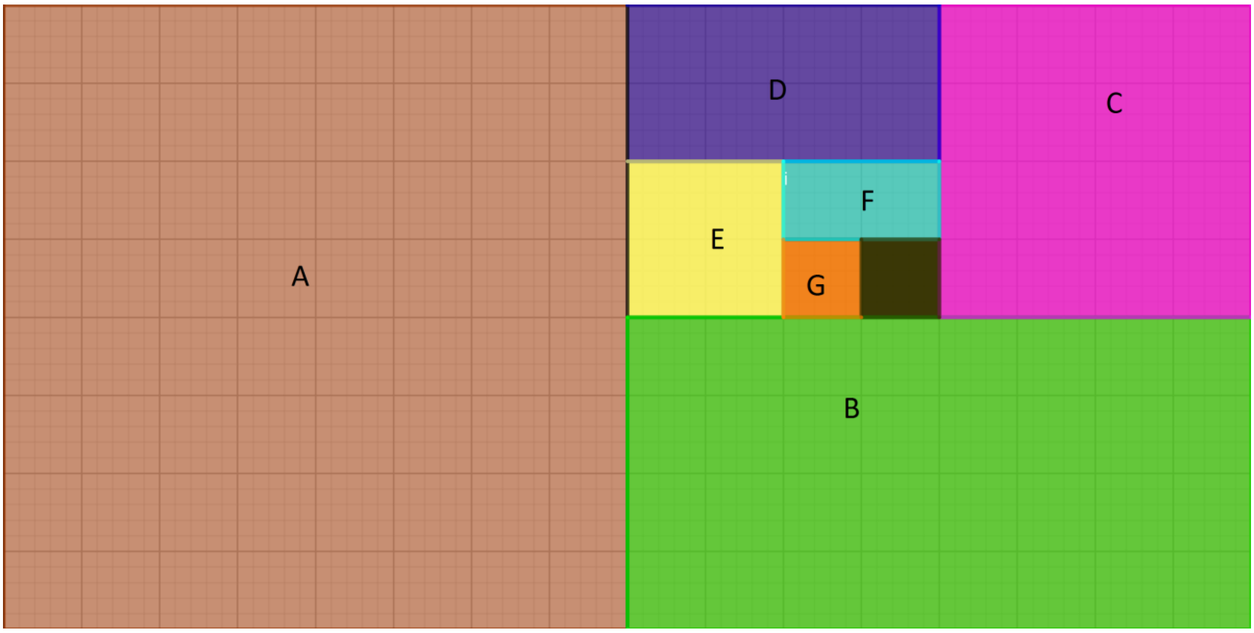
- Student can calculate half of a natural number.
- Sum of infinite terms in geometric sequence.

Material Required :

- Graph
- pencil

Procedure:-

1. Select a natural number x.
2. Find half of that give to some one say A and keep remaining x/2 with you.
3. Find the half of x/2
4. Give x/4 to some other one say B, and keep remaining x/4 with you.
5. Find the half of x/4
6. Give x/8 to some other one say c, and keep remaining x/8with you
7. Find the half of x/8
8. Give x/16 to some other one say D, and keep remaining x/16 with you
9. Find the half of x/16
10. Give x/32 to some other one say E, and keep remaining x/32 with you
11. Find the half of x/32
12. Give x/64 to some other one say F, and keep remaining x/64 with you
13. Now keep it continue till the whole is distributed among individuals.



Observation:-

INDIVIDUAL	PART RECEIVED
A	$\frac{x}{2}$
B	$\frac{x}{2^2}$
C	$\frac{x}{2^3}$
D	$\frac{x}{2^4}$
E	$\frac{x}{2^5}$
………………	
nth. Individual	$\frac{x}{2^n}$

As the part received by individual is written in order :

$\frac{x}{2}, \frac{x}{2^2}, \frac{x}{2^3}, \frac{x}{2^4}, \frac{x}{2^5}, \frac{x}{2^6}, \ldots\ldots\ldots\ldots\ldots\ldots, \frac{x}{2^n}, \ldots\ldots\ldots\ldots$

$\frac{x}{2}, \frac{x}{2} \times \left(\frac{1}{2}\right), \frac{x}{2} \times \left(\frac{1}{2}\right)^2, \frac{x}{2} \times \left(\frac{1}{2}\right)^3, \frac{x}{2} \times \left(\frac{1}{2}\right)^4, \ldots\ldots\ldots\ldots\ldots\ldots, \frac{x}{2} \times \left(\frac{1}{2}\right)^{n-1}, \ldots\ldots\ldots\ldots$

$$First\ Term = a = \frac{x}{2}$$

$$Common\ Ratio = \frac{1}{2}$$

$$S_\infty = \frac{x}{2} + \frac{x}{2} \times \left(\frac{1}{2}\right) + \frac{x}{2} \times \left(\frac{1}{2}\right)^2 + \frac{x}{2} \times \left(\frac{1}{2}\right)^3 + \frac{x}{2} \times \left(\frac{1}{2}\right)^4 + \cdots \ldots\ldots\ldots\ldots\ldots$$

$$S_\infty = \frac{x/2}{1 - \frac{1}{2}} = x$$

Learning outcome:-

Every natural number can be expressed as sum of infinite terms in geometric progression.

Activity : 29

AIM: - Construction of Geometric progression.

Pre-requisite Knowledge:-

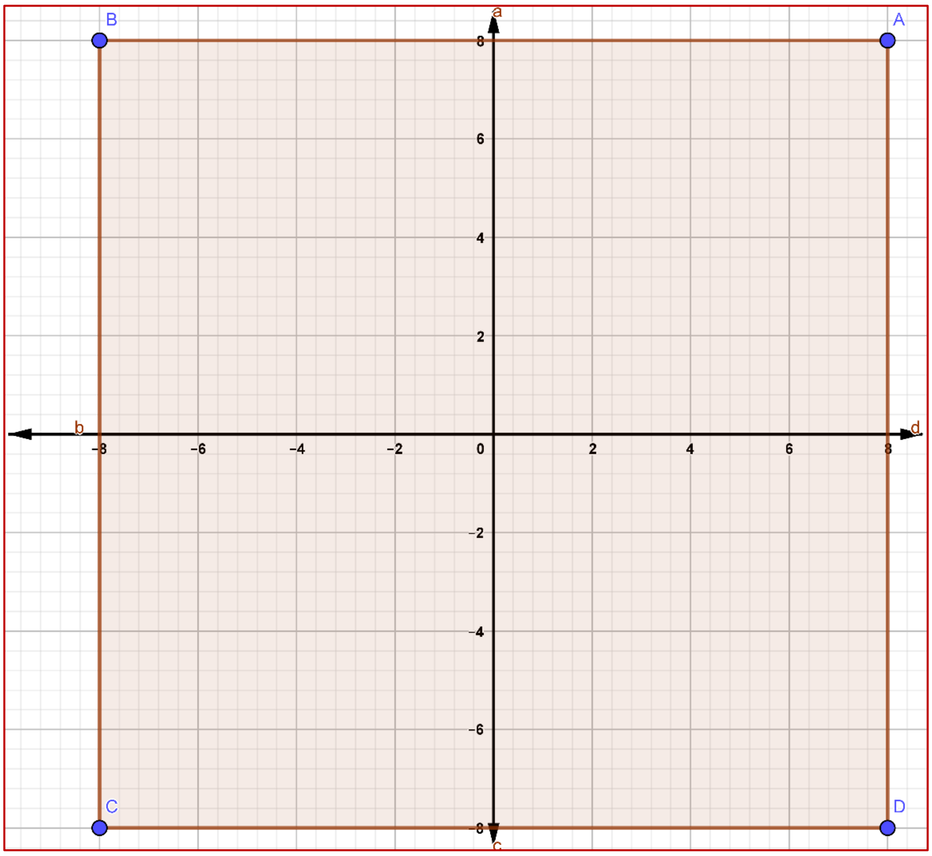
- Square
- Perimeter of square
- Area of square
- Algebraic knowledge of AP and GP.

Material Required:

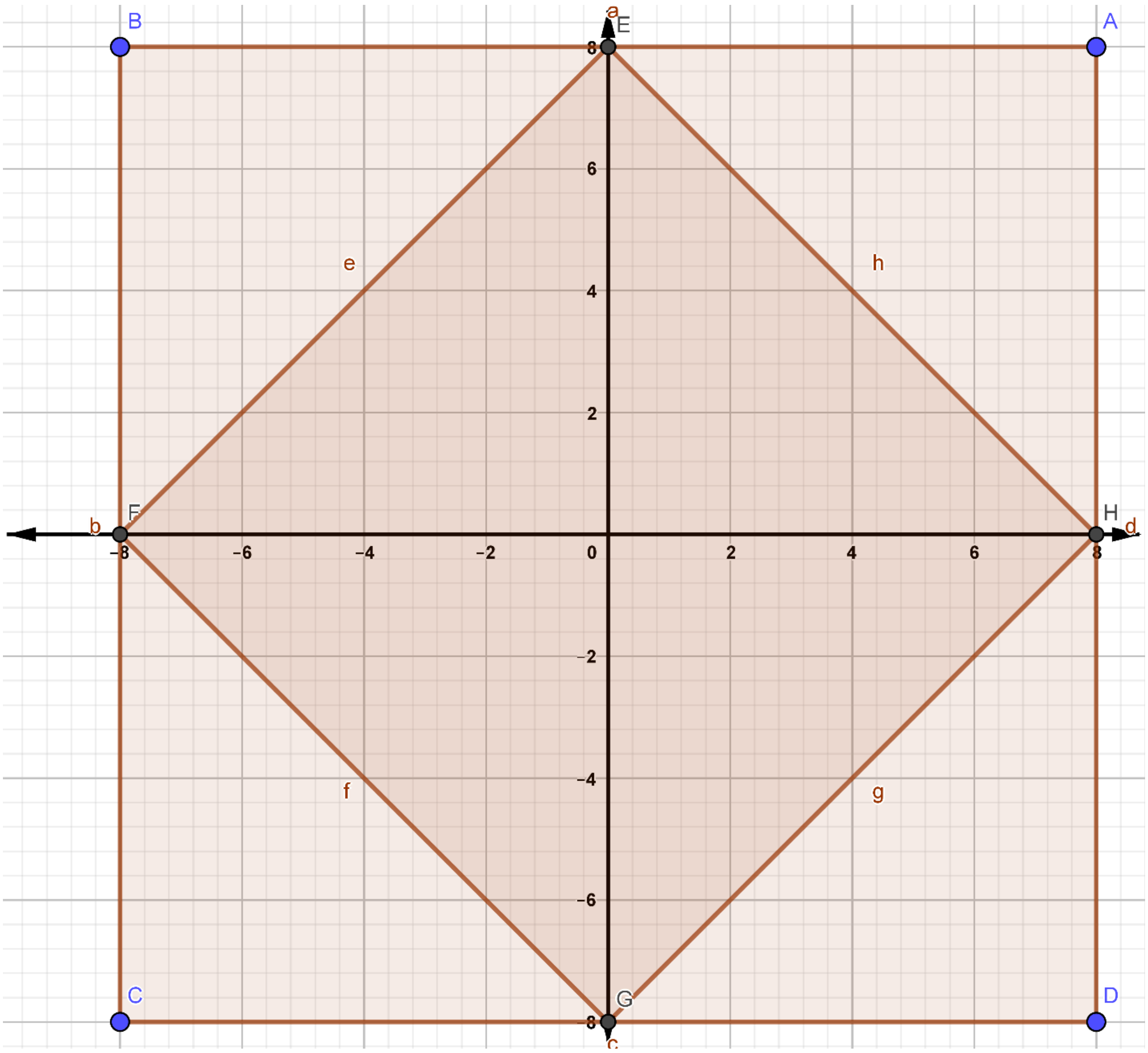
- Geometry box

Procedure:

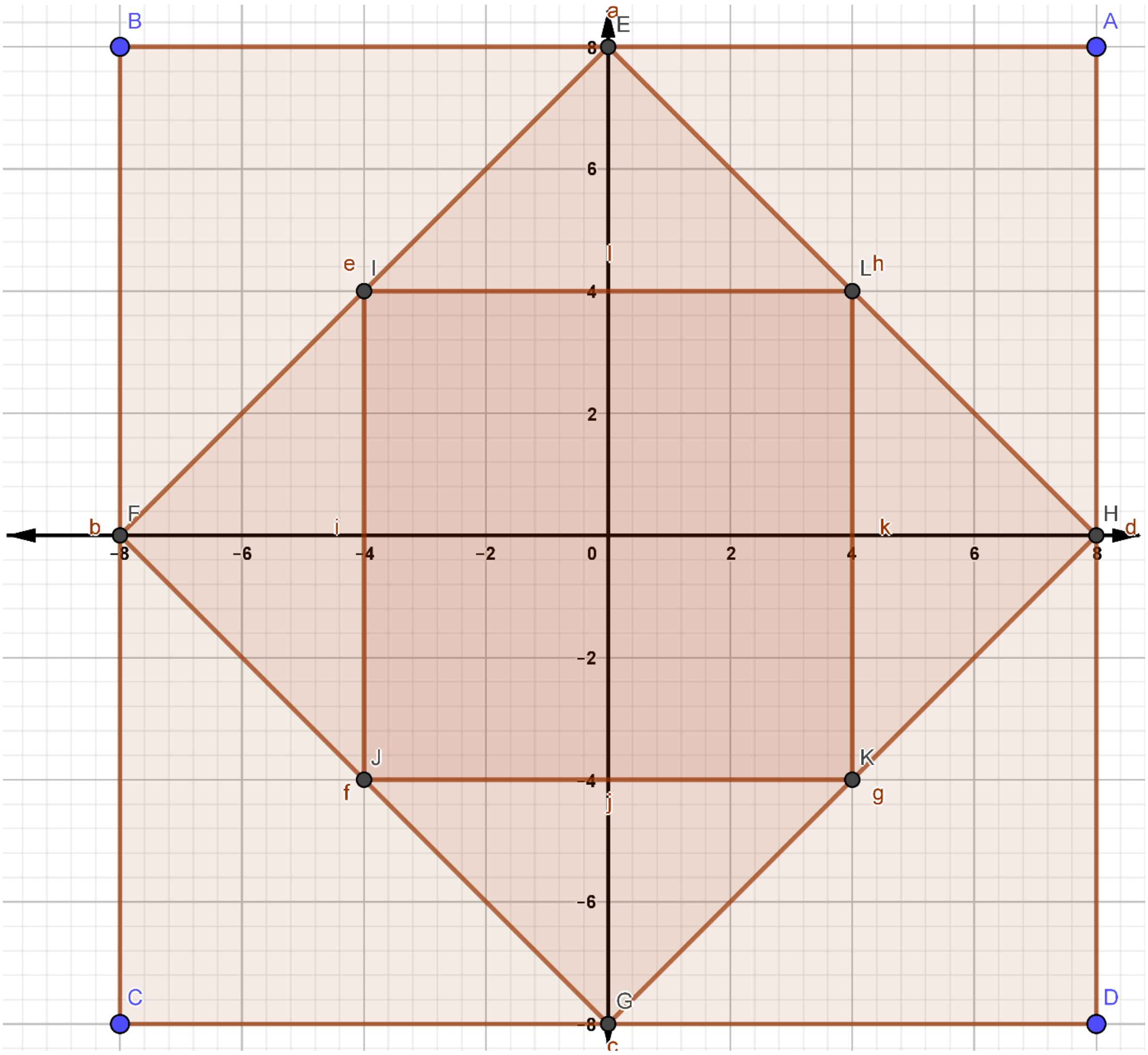
1. Draw a square ABCD of side x unit.



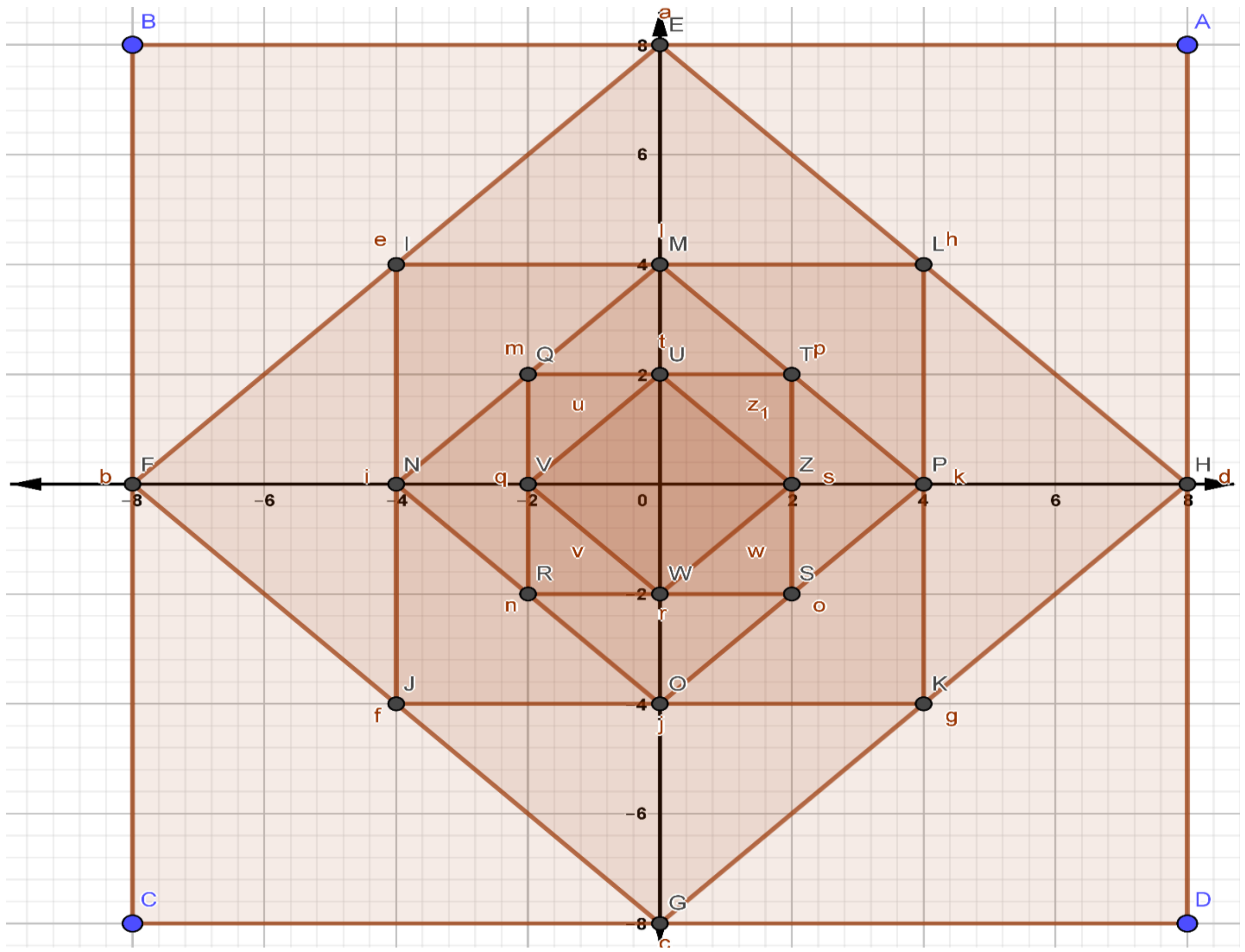
2. Calculate perimeter and area of square ABCD.
3. Locate the mid points of sides AB, BC, CD, and AD.
4. Join these mid points E, F, G and H in order to form a Square EFGH.



5. Find the perimeter and Area of square EFGH.
6. Find the mid points of EF,FG,GHand,HE .
7. Join I, J, K and L in order to form a square IJKL.



8. Find the perimeter and area of square IJKL.
9. Locate the mid points of sides IJ, JK, KL and LI.
10. Join these mid points Of M,N, O and Pin order to form a Square MNOP
11. Find the perimeter and Area of square MNOP
12. Locate the mid points of sides QR, RS, ST and QT.
13. Join these mid points V, W, Z and U in order to form a Square VWZU.
14. Find the perimeter and Area of square VWZU.
15. This can be continued onwards.



Observation:-

SQUARE	PERIMETER	AREA
ABCD (1^{st})	4x	x^2
EFGH (2^{nd})	$4 \times \frac{x}{\sqrt{2}}$	$\left(\frac{x}{\sqrt{2}}\right)^2$
IJKL (3^{rd})	$4 \times \frac{x}{(\sqrt{2})^2}$	$\left(\frac{x}{(\sqrt{2})^2}\right)^2$
MNOP (4^{th})	$4 \times \frac{x}{(\sqrt{2})^3}$	$\left(\frac{x}{(\sqrt{2})^3}\right)^2$
QRST (5^{th})	$4 \times \frac{x}{(\sqrt{2})^4}$	$\left(\frac{x}{(\sqrt{2})^4}\right)^2$
VWZU (6^{th})	$4 \times \frac{x}{(\sqrt{2})^5}$	$\left(\frac{x}{(\sqrt{2})^5}\right)^2$
…………..(n^{th})	$4 \times \frac{x}{(\sqrt{2})^{n-1}}$	$\left(\frac{x}{(\sqrt{2})^5}\right)^2$

Sequence:-

Perimeter: 4x,$4 \times \frac{x}{\sqrt{2}}$,$4 \times \frac{x}{(\sqrt{2})^2}$,$4 \times \frac{x}{(\sqrt{2})^3}$,$4 \times \frac{x}{(\sqrt{2})^4}$,$4 \times \frac{x}{(\sqrt{2})^5}$,……………………..,$4 \times \frac{x}{(\sqrt{2})^{n-1}}$

First Term: 4x

Common Ratio: 1/√2

SUM OF n TERMS:

$$S_n = 4\mathrm{x} + 4 \times \frac{x}{\sqrt{2}} + 4 \times \frac{x}{(\sqrt{2})^2} + 4 \times \frac{x}{(\sqrt{2})^3} + 4 \times \frac{x}{(\sqrt{2})^4} + 4 \times \frac{x}{(\sqrt{2})^5} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. + 4 \times \frac{x}{(\sqrt{2})^{n-1}}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Sum of infinite number of terms

$$S_\infty = \frac{a}{1-r}$$

Area Sequence:x^2,$x^2\left(\frac{1}{2}\right)^1$,$x^2\left(\frac{1}{2}\right)^2$,$x^2\left(\frac{1}{2}\right)^3$,$x^2\left(\frac{1}{2}\right)^4$,$x^2\left(\frac{1}{2}\right)^5$………………,$x^2\left(\frac{1}{2}\right)^{n-1}$

First term: x^2

Common ratio: 1/2

Sum of n terms

$S_n = x^2 + x^2\left(\frac{1}{2}\right)^1 + x^2\left(\frac{1}{2}\right)^2 + x^2\left(\frac{1}{2}\right)^3 + x^2\left(\frac{1}{2}\right)^4 + x^2\left(\frac{1}{2}\right)^5 + \ldots\ldots\ldots\ldots\ldots\ldots + x^2\left(\frac{1}{2}\right)^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Sum of infinite number of terms

$$S_\infty = \frac{a}{1-r}$$

Learning outcome:-

1. Students develop a geometrical intuition for the construction of geometric progression starting from simple area of a square.
2. Students understand area of n terms in geometric progression.
3. Student understand sum of infinite terms in geometric progression.

UNIT : 3

CHAPTER 10 : STRAIGHT LINE

Activity : 30

AIM:- To convert a general equation of a straight line into different forms algebraically and its verification graphically.

Pre-requisite Knowledge:-

- Different forms of a straight line
- Graph of a straight line

Materials required:-

- Graph
- pencil

Procedure:-

14. Take an equation $ax + by + c = 0$ as a general equation of a straight line.
15. Convert this equation in to following forms;
16. Slope intercept form
17. Intercept form
18. Normal form
19. Now parameters in different form are calculated mathematically.
20. Draw the graph of these three forms.
21. Now parameters in different form are calculated graphically.

Observation:-

Conversion of general equation of straight a line into slope intercepts form:

$$General\ equation\ of\ a\ straight\ line\ Ax + By + C = 0, ;provided\ a\ and\ b\ are\ not\ simultaniously\ 0.$$

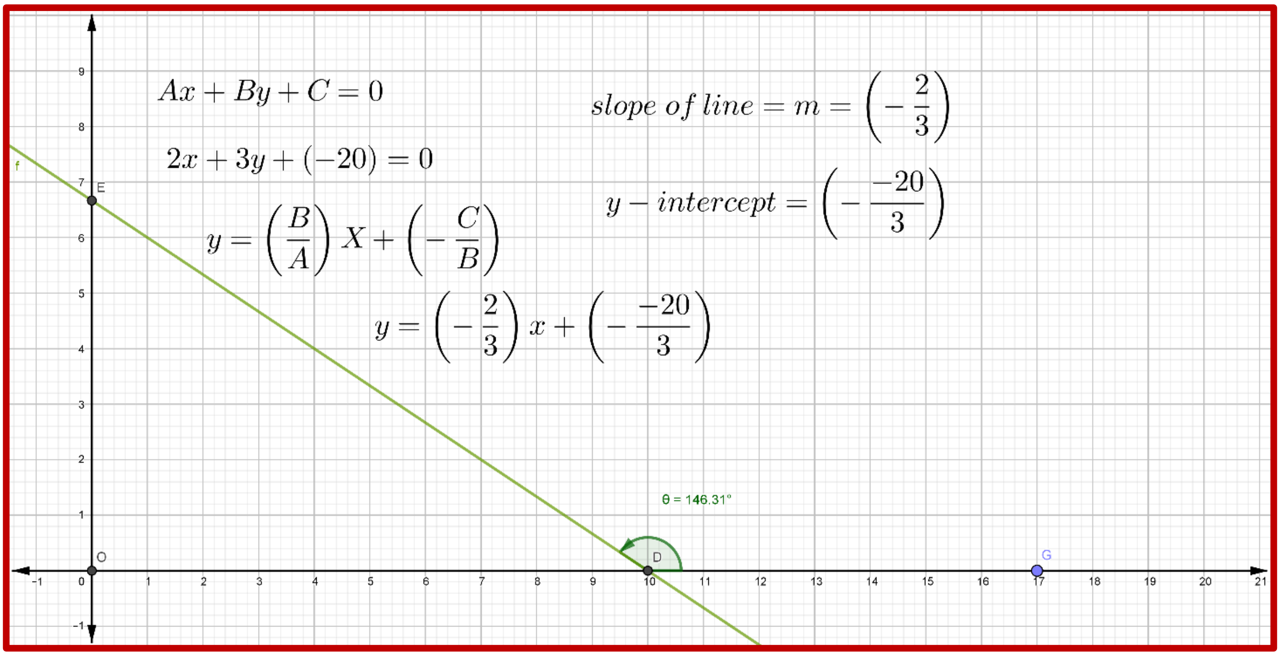
Now this equation can be written in the form:

$$y = \left(-\frac{A}{B}\right)x + \left(-\frac{C}{B}\right)$$

Compare it with $y = mx + c$

$$slope\ of\ the\ line = m = -\frac{A}{B}$$

$$y - intercept = -\frac{C}{B}$$



	ALGEBRAICALLY	FROM GRAPH
SLOPE (M)	$-\frac{A}{B}$	$tan\theta$
Y-INTERCEPT (C)	$-\frac{C}{B}$	LENGTH OE

Conversion of general equation of straight a line into intercept form:

$$General\ equation\ of\ a\ straight\ line\ Ax + By + C = 0, ; provided\ a\ and\ b\ are\ not\ simultaniously\ 0.$$

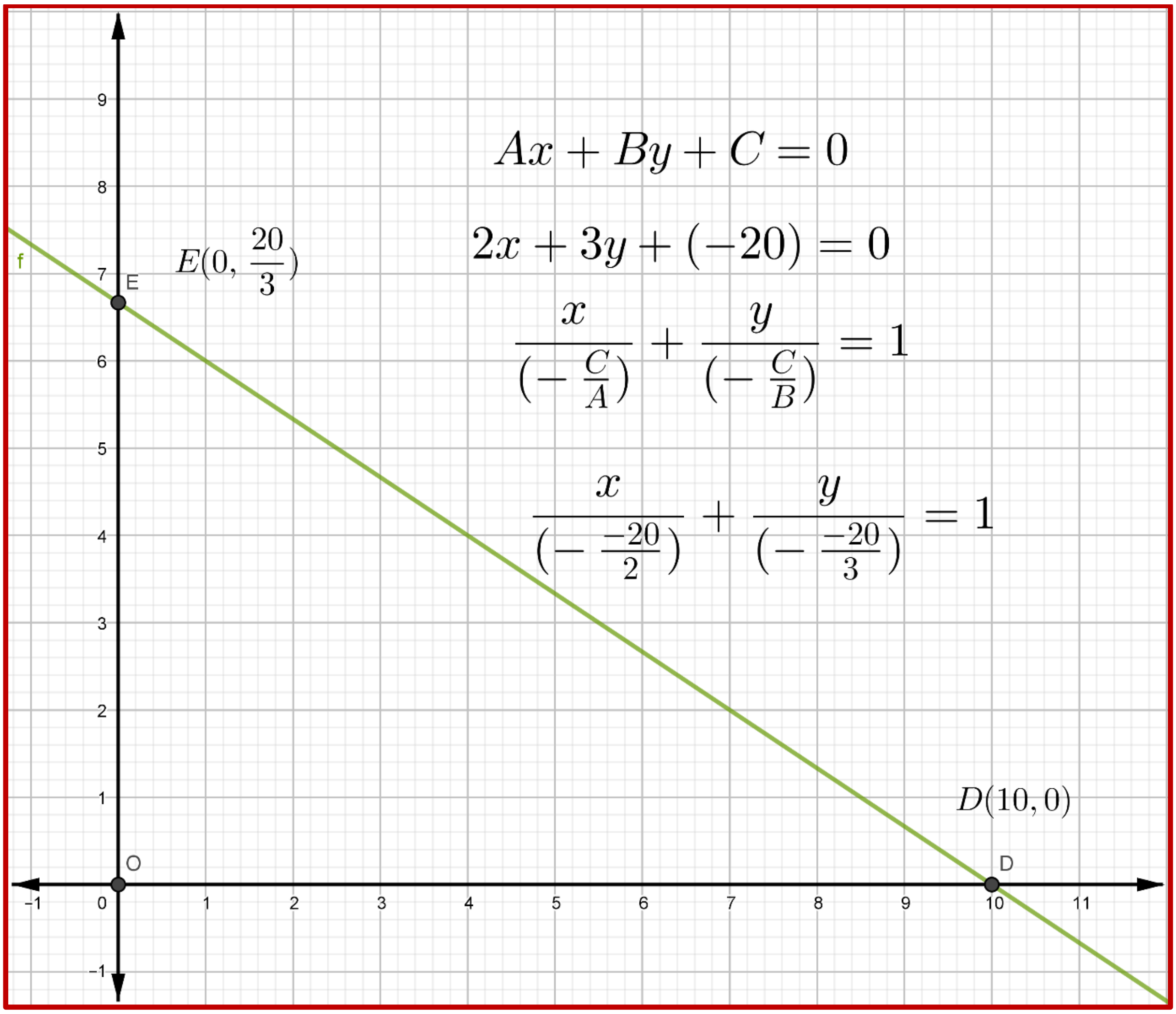
Now this equation can be written in the form:

$$\frac{x}{(-C/_A)} + \frac{x}{(-C/_B)} = 1$$

Compare it with: $\frac{x}{a} + \frac{y}{b} = 1$

$$x - \text{intercept} = \text{a} = \frac{-C}{A}$$

$$Y - \text{intercept} = \text{a} = \frac{-C}{B}$$



	ALGEBRAICALLY	FROM GRAPH
X-INTERCEPT (a)	$-\frac{C}{A}$	LENGTH OD
Y-INTERCEPT (b)	$-\frac{C}{B}$	LENGTH OE

Conversion of general equation of straight a line into Normal form:

$$General\ equation\ of\ a\ straight\ line\ Ax+By+C = 0,;provided\ a\ and\ b\ are\ not\ simultaniously\ 0.$$

Now this equation can be written in the form:

$$Ax+By=-C$$

$$compare\ it\ with\ x\times cos\omega+y\times sin\omega = \text{p}$$

$$\frac{A}{cos\omega}=\frac{B}{sin\omega}=\frac{-C}{p}$$

$$this\ gives\ cos\omega = -\frac{Ap}{C}\ and$$

$$sin\omega = -\frac{Bp}{C}$$

- Now $cos^2\omega + sin^2\omega = 1$ $\qquad\qquad \left(-\frac{Ap}{C}\right)^2 + \left(-\frac{Bp}{C}\right)^2 = 1$

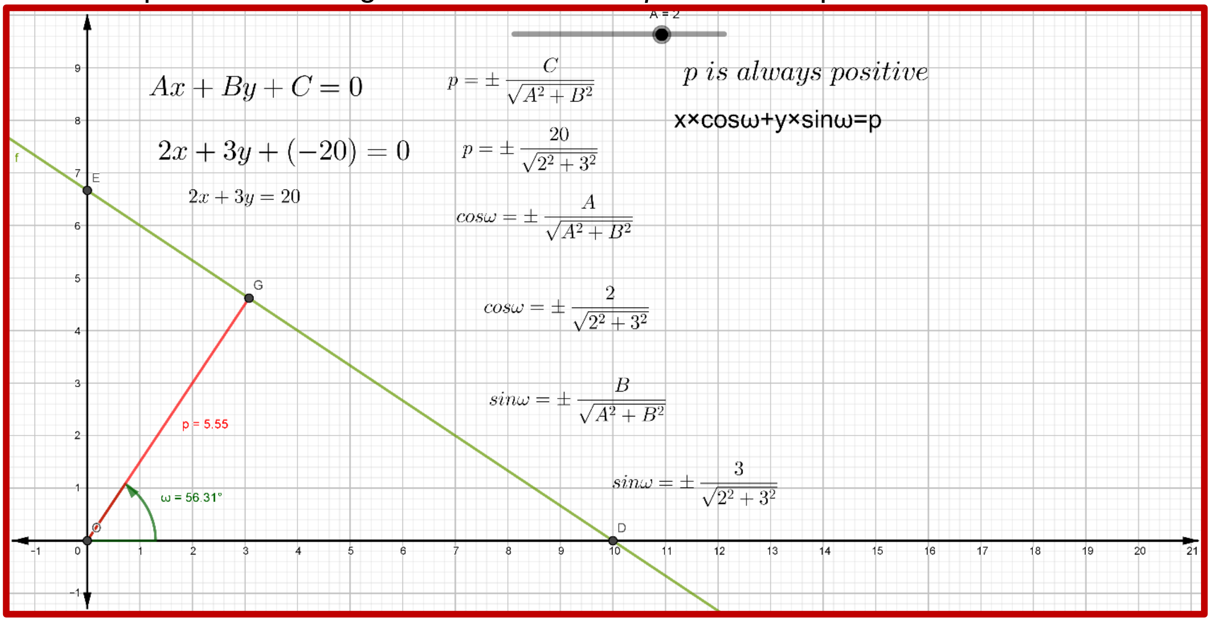
$$p^2 = \frac{C^2}{A^2 + B^2}$$

$$p = \pm\frac{C}{\sqrt{A^2 + B^2}}$$

$$this\ gives\ cos\omega = \pm\frac{A}{\sqrt{A^2 + B^2}}\ and$$

$$sin\omega = \pm\frac{B}{\sqrt{A^2 + B^2}}$$

•Proper choice of signs is made so that *p* should be positive.



	ALGEBRAICALLY	FROM GRAPH
Length perpendicular drawn from origin to line = p	$-\frac{C}{A}$	LENGTH OG
Angle between perpendicular And x-axis	$\tan^{-1}\frac{B}{A}$	ω

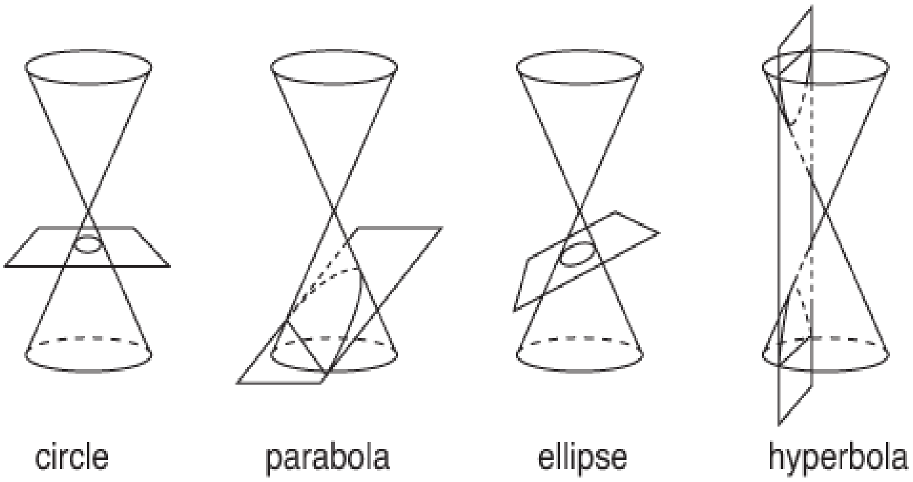
Learning outcome:-

Conversion of General equation of a straight line is verified graphically.

CHAPTER 11-CONIC SECTIONS

Activity -31

AIM:- Formation of conic sections

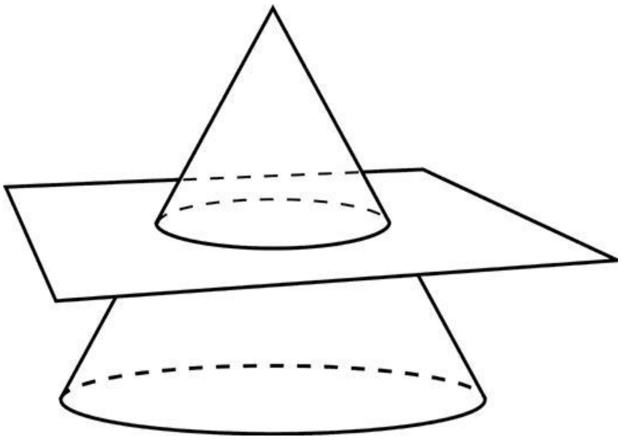


MATERIALS REQUIRED:-

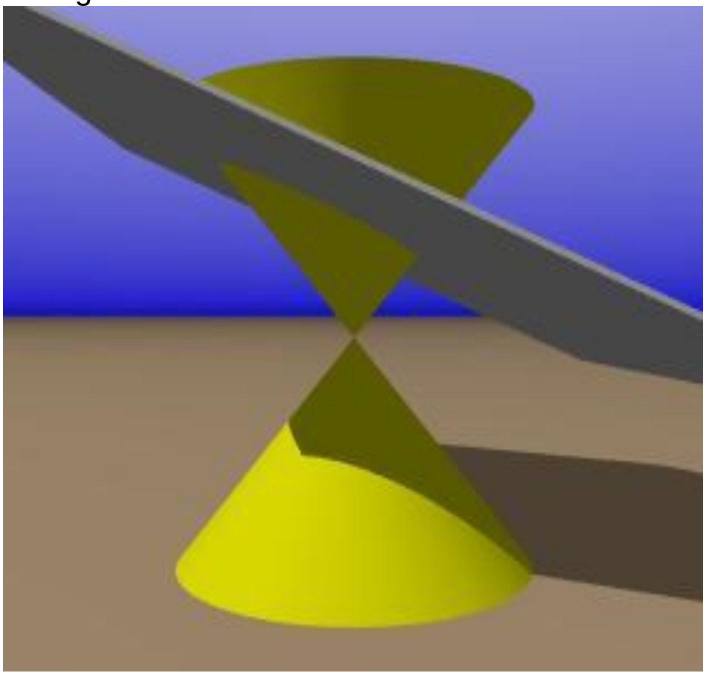
1. Hardboard
2. Scissors
3. Adhesive
4. Transparent sheet
5. Geometry box

PROCEDURE:-

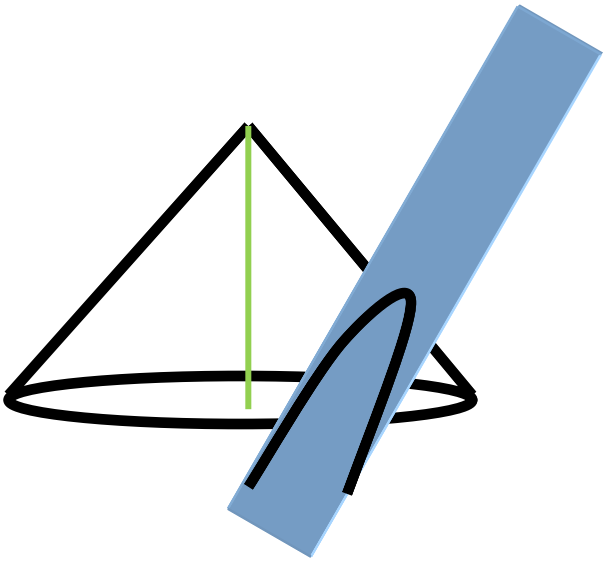
1) Cut two transparent sheets in the form of a sector of circle.
2) Form two right circular cones by folding the transparent sheet.
3) Join the tips of cones to each other and fix the double napped cone on hardboard.
4) Now cut the cone by plane sheet(blade/knife) such that plane sheet is perpendicular to the axis of the cone,then the section will be a "CIRCLE".



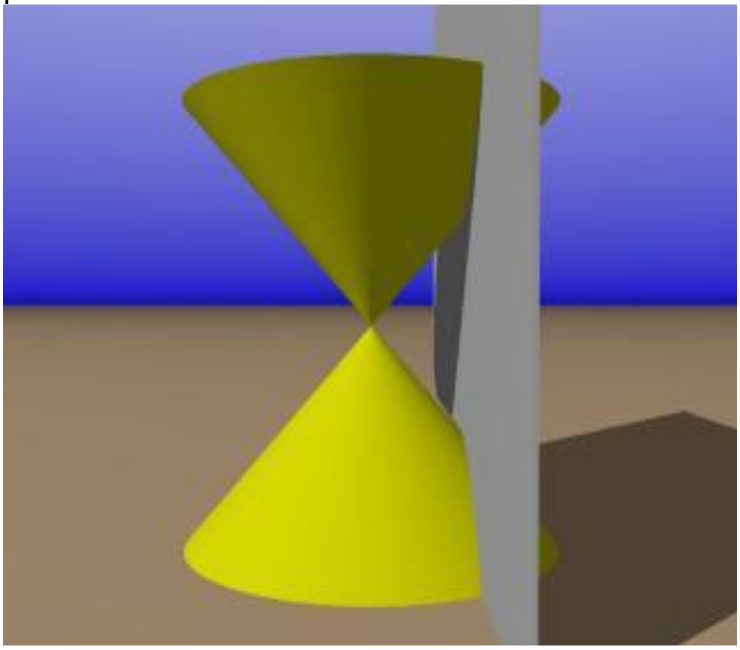
5) Cut the cone by a plane such that the plane is slightly inclined to the axis of the right circular cone.The section will be an “ELLIPSE”.



6) Cut the cone by a plane such that the plane is parallel to the generator of the cone. The section will be a “PARABOLA”.



7) Cut the double napped right circular cone by a plane such that the plane is parallel to the axis of the cone.The section will be a “HYPERBOLA”.



Conclusions:-

- This activity explains different types of conic sections.
- Model can be used in visualizing the conic sections.

Activity- 32

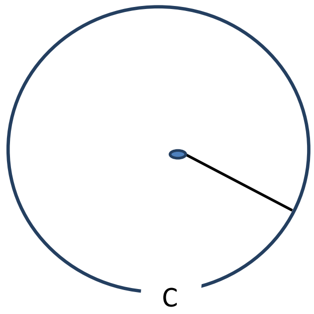
AIM:- Formation of a circle.

MATERIALS REQURIED:-

1. Hard board
2. Chart paper
3. Pencil
4. Nail
5. Adhesive

PROCEDURE:-

1. Paste a chart paper on the hard-board.
2. Fix a nail at the centre of the hard board.
3. Tag one end of thread with nail and another end of the thread with the pencil.
4. Now rotate the pencil around the nail with the stretched position of the thread .



CONCLUSION:-

The figure thus form is a circle

LEARNING OUTCOMES:- It gives the explanation of formation of circle.

Activity -33

AIM: Construction of Parabola.

MATERIALS REQUIRED: -

1. Hardboard
2. Colored chart paper
3. Nails
4. Hammer
5. Markers
6. Pencil
7. Scale
8. Protractor
9. Nylon wire

PROCEDURE:-

1. Take a hardboard of size 30cmx30cm.
2. Cover it with coloured chart paper.
3. Draw two line segments AB and AC each of length 20cm,inclined each other at an angle of 35^0.

4.Divide AB and AC into 15 equal parts.

5. Name the points of division along AB from 1 to 15 and the points of division

Along AC from 15 to 1.

6.At each numbered division points,fix the nails.

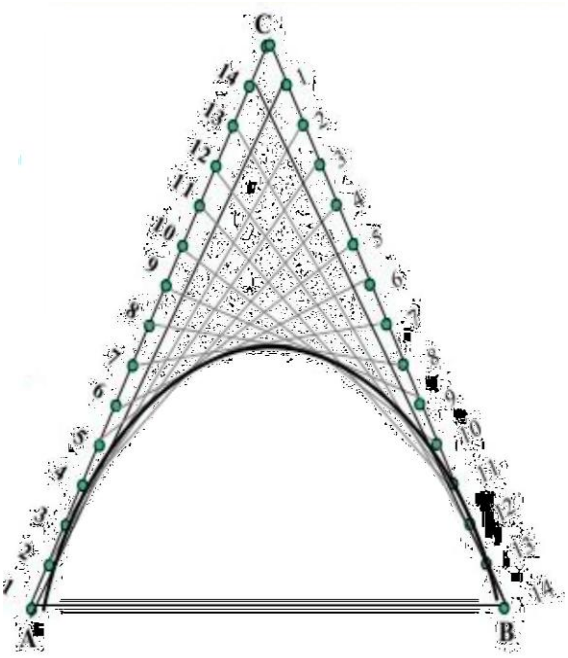
7.Join together the equally marked points of division.

i.e.1 is joined with 1 ,

2 is joined with 2,

3 is joined with 3 and so on

15 is joined with 15 with nylon wire.



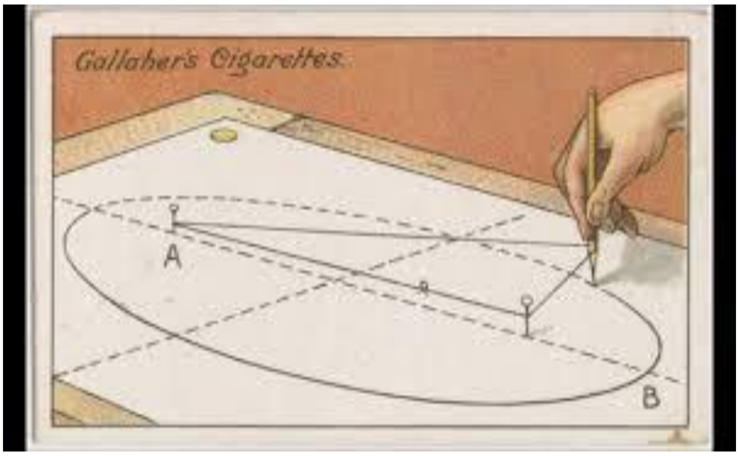
Conclusion: The figure thus formed is a ‘PARABOLA’.

LEARNING OUTCOMES:-

This activity can be used to explain the construction of PARABOLA.

Activity -34

AIM:- Formation of ellipse.

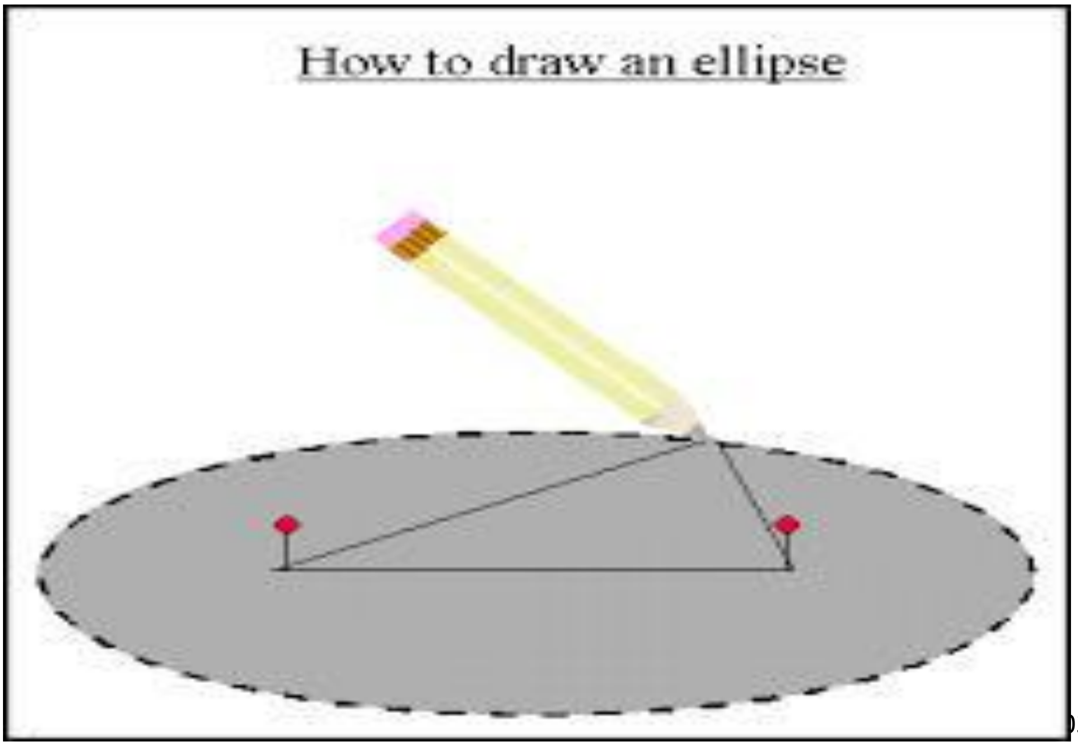


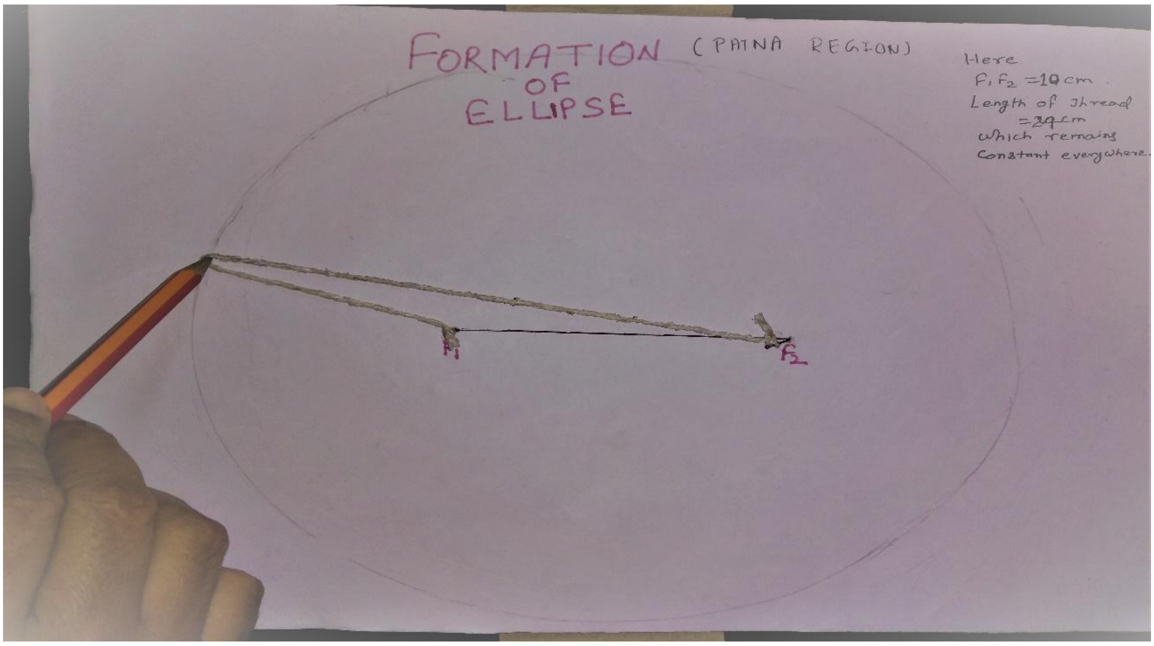
MATERIALS REQUIRED:-

1. Hardboard
2. Chart paper
3. Nails
4. Adhesive
5. Pencil
6. Scale
7. Thread

PROCEDURE:-

4. Take a hardboard of size 30cmx30cm.
5. Cover it with chart paper
6. Fixed two nails at a distance of 7cm near the centre of the board.
7. Take a thread 0f 20 cm and tag it with two nails.
8. Tag a pencil with thread.
9. Rotate the pencil on the plane chart paper such that the sum of distances from two fixed points is always constant.
10. The figure thus formed is an “ELLIPSE”.





LEARNING OUTCOMES:

- With this activity we can visualize the Ellipse.
- It explains the ellipse and derivation of an equation of ellipse may be done easily.

CHAPTER 12 THREE-DIMENSIONAL GEOMETRY

Activity - 35

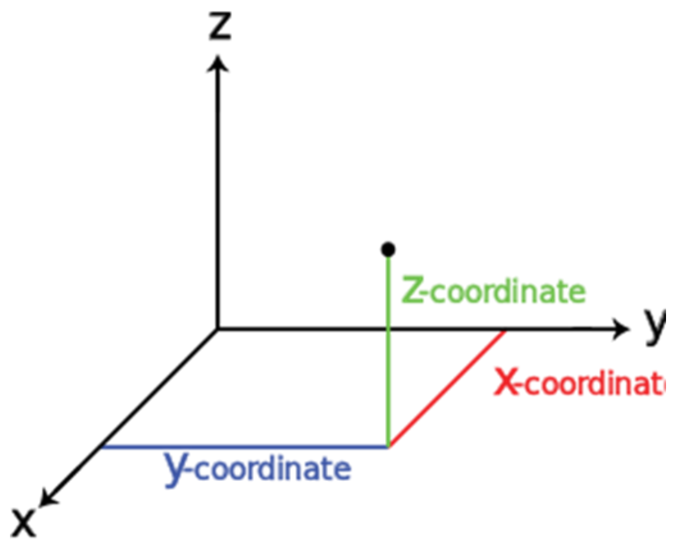
AIM: To find out distance between two points in Space

MATERIALS REQUIRED -

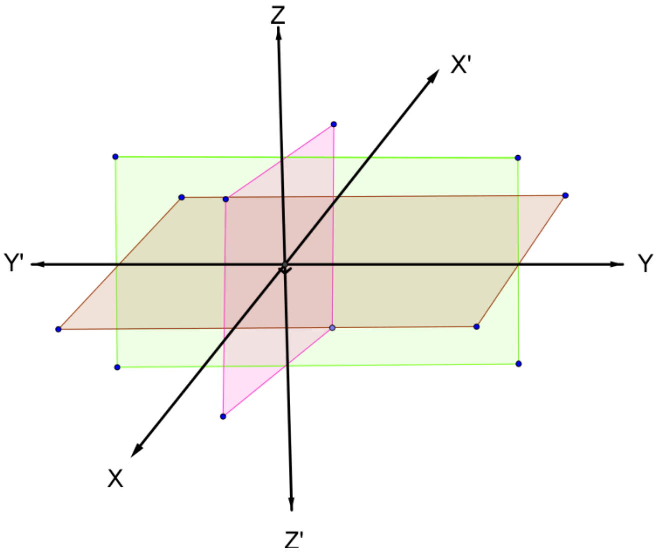
- Three cardboards each of size 30cm x 30cm.
- Nails of varying lengths with caps on one end.
- Wire or thread.
- Glazing Papers.
- Scale and pencil.

PROCEDURE-

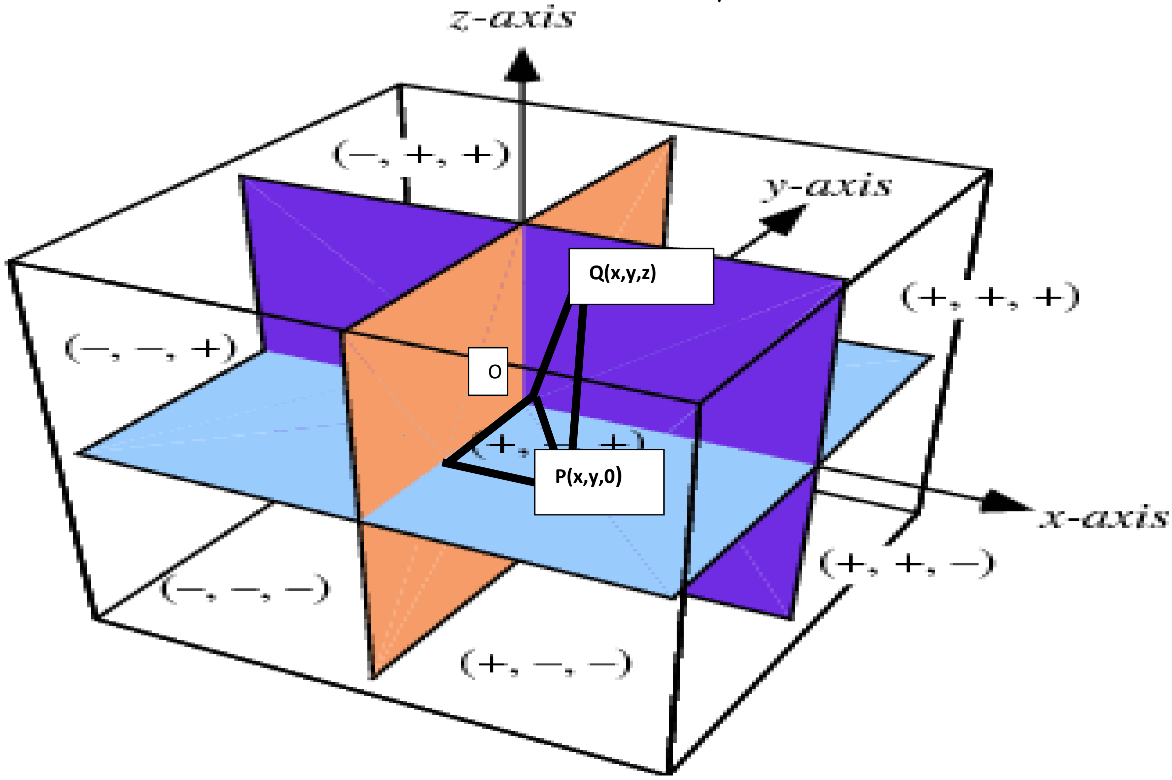
1. Take three sheets (thin) of cardboards each of size 30cm x 30 cm.
2. Adjust two sheets in such a way that they intersect orthogonally in the middle of each other.
3. Cut the third sheet into two equal rectangles.
4. Insert one rectangle from one side in the middle cutting the two orthogonally and the other rectangle from other side.
5. These three planes intersecting each other at right angles at a point 'O' and they divide the space into eight parts.
   Each part is called an **Octant.**
6. In octants fix scale to show X-axis, Y-axis and Z-axis. The other sides are represented as XX', YY' and ZZ'.

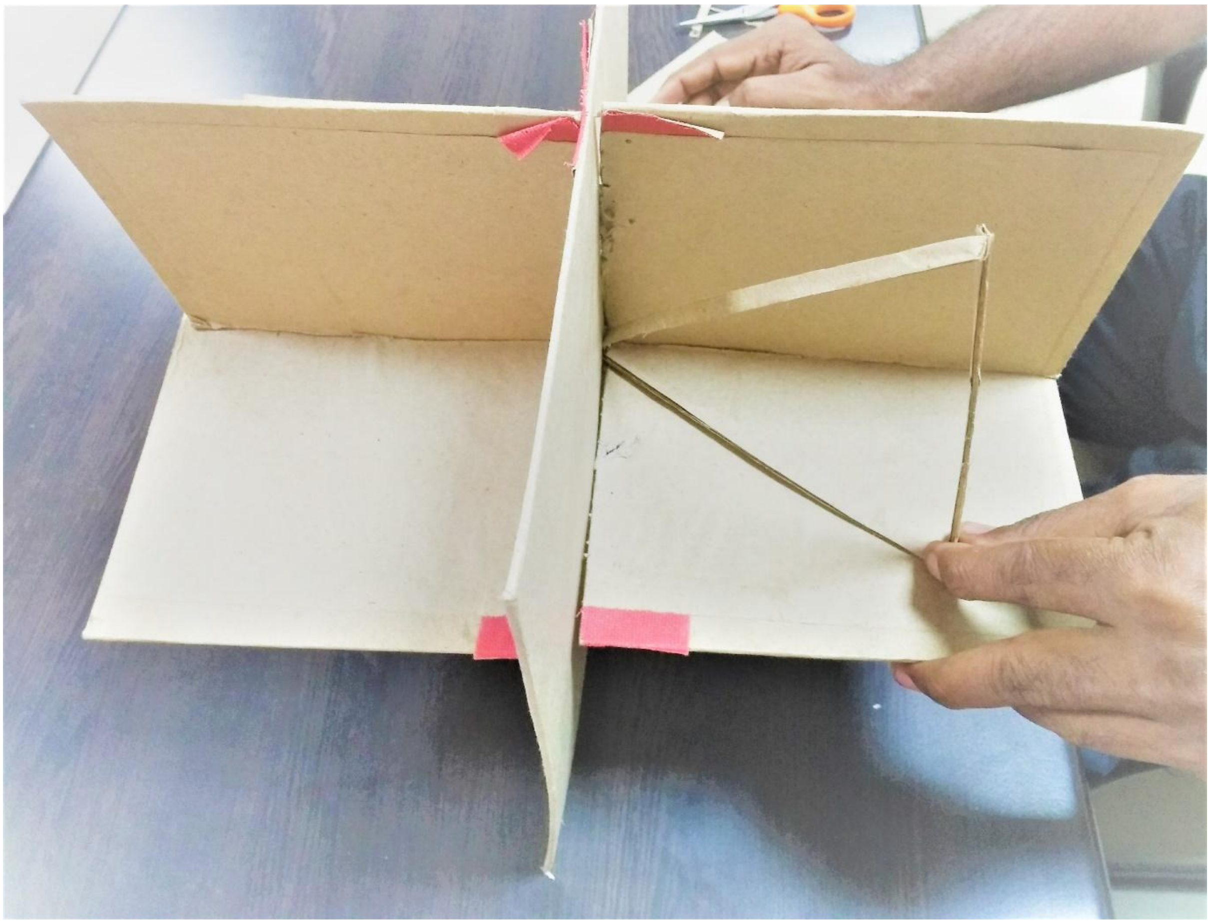


7. We take XOY plane on plane of paper and Z'OZ plane as perpendicular To the plane of XOY.



8. Fix a nail on XY plane and named it as PQ, where P is on the XY-plane and Q in the space.
9. The distance of point P on the XY-plane with coordinates (x, y) from the origin (0, 0) is $\sqrt{x^2+y^2}$.
10. Since nail PQ is perpendicular on XY-plane and parallel to Z-axis.
11. Join origin to upper tip Q of PQ.
12. Clearly coordinates of Q be (x, y, z) in space.
13. Thus distance of Q(x, y, z) from the origin O (0, 0,0) is $\sqrt{x^2+y^2+z^2}$.





LEARNING OUTCOMES:-

- The activity visualizes the position of a point in space.
- We can find the distance of a point in space from the origin.

UNIT IV: CALCULUS

CHAPTER 13: LIMITS AND DERIVATIVES

Activity- 36

Aim:- To find the value of $\lim_{x \to a} f(x)$

Material required:- Paper, calculator, graph paper etc.

Example: Find the value of $\lim_{x \to 3} \frac{x^2-9}{x-3}$

Procedure:-

- Write x= 3-h where h is the smallest increment such that h> o and find f(x).
- Write x=3+h where h is the smallest increment such that h> o and find f(x).
- Plot the graph.
- Observe the value of f(x) at 3-h, 3+h and x=3.

Worksheet:-

x	2.4	2.5	2.6	2.7	2.8	2.9	2.99	3.01	3.1	3.2	3.3	3.4	3.5	3.6
f(x)	5.4	5.5	5.6	5.7	5.8	5.9	5.99	6.01	6.1	6.2	6.3	6.4	6.5	6.6

Observation:- When x→ 3^- and x→ 3^+ the value f(x)→ 6

Conclusion:-- As x→ 3 the value f(x)→ 6

Activity- 37

Aim:- To evaluate one side limit (left hand and right hand limit).

Material required:- Paper, calculator, graph paper etc.

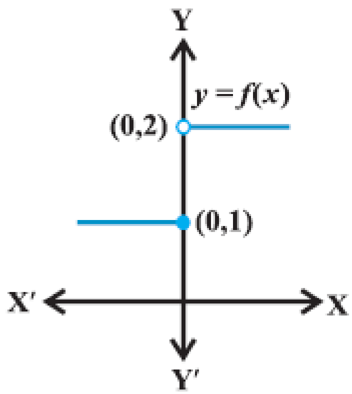
Example: $f(x) = \begin{cases} 1, & x \leq 0 \\ 2, & x > 0 \end{cases}$

Procedure:-

1. Plot f(x) for $x \leq 0$ and$x > 0$.
2. Write *left hand* and *right hand* limits.
3. Plot the graph using worksheet.
4. Compare *left hand* and *right hand* limits of the function.

Worksheet:-

x	$\mathbf{-0.1}$	$\mathbf{-0.01}$	$\mathbf{-0.001}$	$\mathbf{-0.0001}$	0	1.0001	1.0001	1.0001	1.0001
h(x)	1	1	1	1	**1**	**2**	**2**	**2**	**2**



Observations:-

Left hand and right hand limits are not equal.

Conclusion:- Limit does not exist when Left hand and right hand limits are not equal.

Activity- 38

Aim:- To evaluate the limit of given function

Material required:- Paper, calculator, graph paper etc.

Example:- Find the limit of the function f, f(x) = x+10 at x = 5.

Procedure:-

- Take x= 5-h where h is the smallest increment such that h> o and find f(x).
- Take x= 5+h where h is the smallest increment such that h> o and find f(x).
- Plot the graph.

Work sheet:-

x	4.9	4.95	4.99	4.995	5.001	5.01	5.1
f(x)	14.9	14.95	14.99	14.995	15.001	15.01	15.1

Observation:-

- We observe that value of *f*(*x*) at *x* = 5 should be greater than14.995 and less than 15.001 assuming nothing dramatic happens between *x* = 4.995and 5.001.
- It is reasonable to assume that the value of the *f*(*x*) at *x* = 5 as dictated by the numbers to the left of 5 is 15, i.e. $\lim_{x\to 5-} f(x) = 15$
- Similarly, when *x* approaches 5 from the right, *f*(*x*) should be taking value 15, i.e., $\lim_{x\to 5+} f(x) = 15$

Conclusion:-

$$\lim_{x\to 5-} f(x) = \lim_{x\to 5+} f(x) = 15$$

UNIT -V: MATHEMATICAL REASONING

CHAPTER 14: MATHEMATICAL REASONING

Activity 39

Aim:- To check whether given sentence is statement or not.

Material required:- Chart paper, Pencil, Scale, sketch- pens, data of sentences, etc.

Procedure: -

1. Choose the sentences from different sources.
2. Teacher divides the students into two groups and distributes work sheet to each student.
3. Teacher asks students to complete the worksheet

Worksheet:-

Sr.No.	Sentence	True/False /ambiguous	Is statement? (Yes/Not)
1.	Two plus two equals four.		
2.	The sum of two positive numbers is positive.		
3.	All prime numbers are odd numbers.		
4.	How beautiful!		
5.	Open the door.		
6.	Where are you going?		
7.	In 2003, the president of India was a woman.		
8.	An elephant weighs more than a human being.		
9.	Women are more intelligent than men.		
10	She is a mathematics graduate.		
11.	Kashmir is far from here.		
12.	Tomorrow is Friday.		

Observation:-

From above worksheet we observe that some of the sentences are true or false. Such sentences are acceptable as a statement in mathematics.
Some sentences are ambiguous. Such a sentences are not acceptable as a statement in mathematics.

Conclusion:-

A sentence is mathematically acceptable statement if it is either true or false but not both.

Activity - 40

Aim:- To state whether the given compound statement is true or false by connecting word “or”.

Material required:- Card board, switches, wire, bulbs, battery etc.

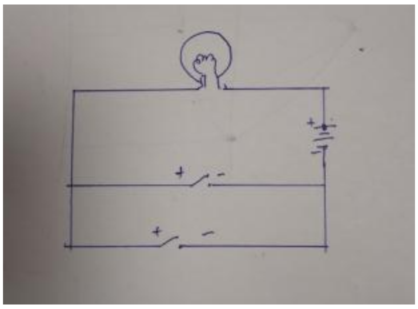
Procedure: -

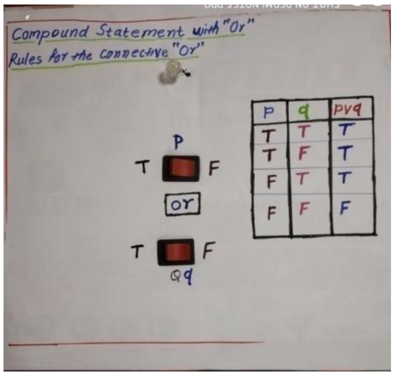
1. Read a compound statement
2. If component statement is true then write T.
3. If component statement is false then write F.

Demonstration:-

1. If statement is true (T) then press switch at T
2. If statement is false (F) then press switch at F.

Circuit diagram.





Observation:-

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

Conclusion:-

1. A compound statement with an ‘Or’ is true when one component statement is true or both the component statements are true.
2. A compound statement with an ‘Or’ is false when both the component statements are false.

<u>**Activity- 41**</u>

Aim:- To state whether the given compound statement is true or false by connecting word “and”.

Material required:- Card board, switches, wire, bulbs, batteries etc.

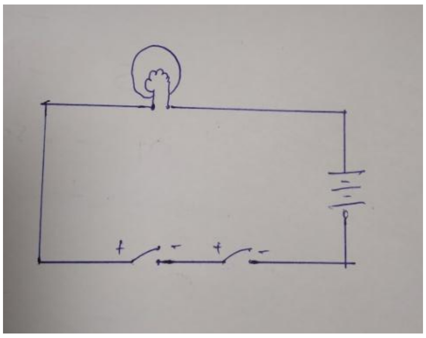
Procedure:-

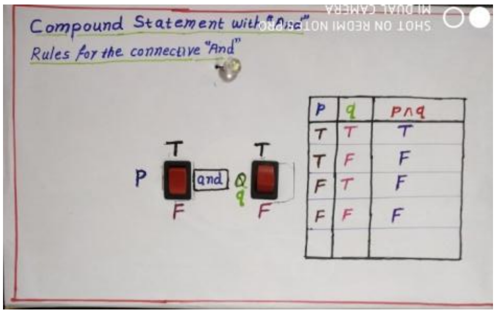
1. Read a statement
2. If statement is true then write T.
3. If statement is false then write F.

Demonstration:-

1. If statement is true then press the switch at T.
2. If statement is false then press the switch at F.

Circuit diagram.





Observation:-

p	q	p $\wedge$ q
T	T	T
T	F	F
F	T	F
F	F	F

Conclusion:-

1. The compound statement with ‘And’ is true if all its component statements are true

2. The component statement with ‘And’ is false if any of its component statements is false.

UNIT -VI: STATISTICS AND PROBABILITY

CHAPTER 15:- STATISTICS

Activity - 42

Aim:-To analyse the observation for rolling a pair of dice, noting the sum of the numbers of dots on the upper faces two dice and interpret the data.

Material Required:- Chart Paper, Sketch Pen of different Colours, a pair of dice.

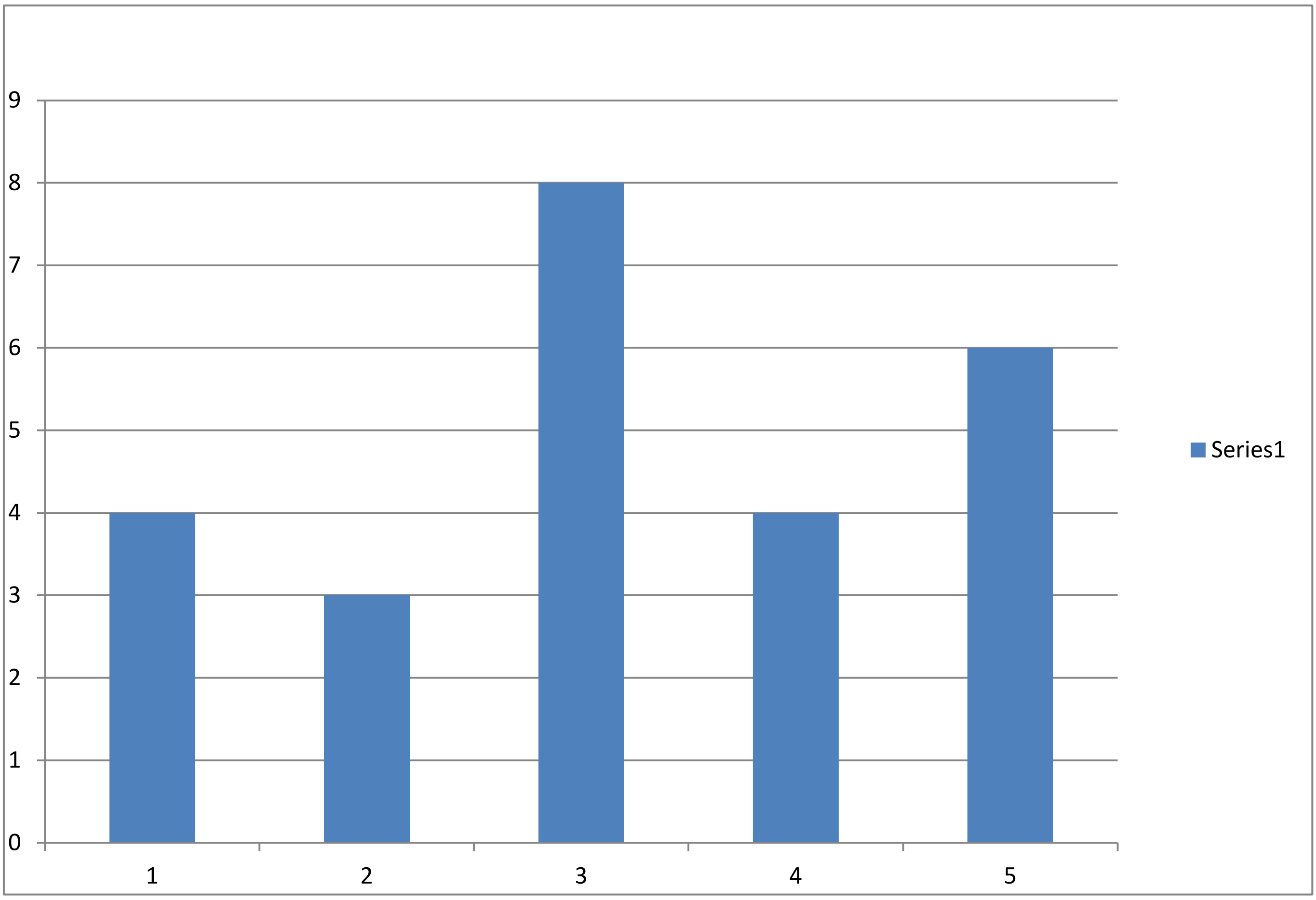
Procedure:-

1. A pair of dice is rolled and observations are noted down.
2. We represent the data graphically and find mean and mean deviation of the data.
3. Interpretation of the data.
   Demonstration:-
   1. A pair of dice is thrown 25 times.
   2. Observations are noted down.

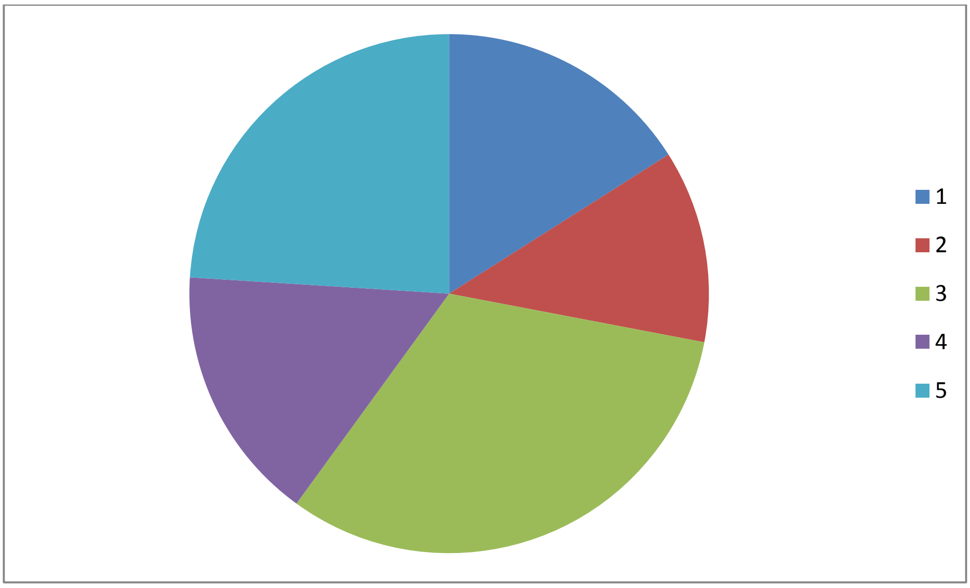
Sum of nos. dots on top faces(x_i)	No. of times(f_i)
2	4
6	3
8	8
10	4
12	6

4.Diagrammatic Representation of data.

SL.No.	Sum of nos. of dots on top faces(x_i)	No. of times(f_i)
1	2	4
2	6	3
3	8	8
4	10	4
5	12	6



Bar Graph for sum of numbers of dots on the upper faces



Pie Chart for sum of numbers of dots on the upper faces

Here, 1:- Sum of nos. on the tops of two dice is 2

2:- Sum of nos. on the tops of two dice is 6

3:- Sum of nos. on the tops of two dice is 8

4:- Sum of nos. on the tops of two dice is 10

5:- Sum of nos. on the tops of two dice is 12

Calculation of Mean Deviation from the Mean.

Sum of nos. on top faces(x_i)	No. of times(f_i)	$x_i f_i$	$f_i\lvert x_i - \bar{x}\rvert$
2	4	8	24.32
6	3	18	6.24
8	8	64	0.64
10	4	40	7.68
12	6	72	23.58
Total	$\sum fi = 25$	$\sum f_i x_{i=} 202$	$\sum f_i\lvert x_{i-}\bar{x}\rvert$=62.4

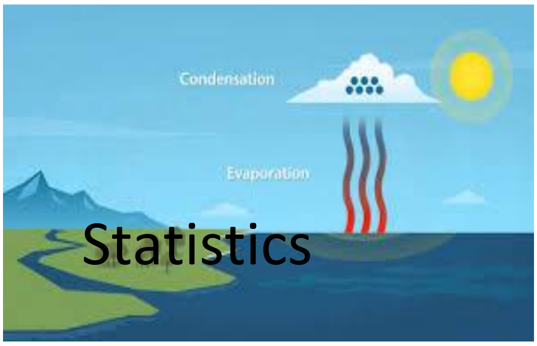
Mean=$\bar{x}$ =$\frac{\sum f_i x_i}{\sum f_i} = \frac{202}{25} = 8.08$

Mean Deviation ($\bar{x}$) =$\frac{\sum f_i |x_{i-}\bar{x}|}{\sum f_i} = \frac{62.8}{25} = 2.496$

Interpretation of Data

From the analysis of the data it is clear that mean is concentrated about the value 8.

Mean deviation from the mean is large, hence data is widely scattered.



Activity-43

Aim:- To collect average weather condition for a year of a city. Represent the data graphically, analyse the data and give interpretation.

Material Required:- Chart Paper, Sketch Pen of different Colours, collection of data from various sources.

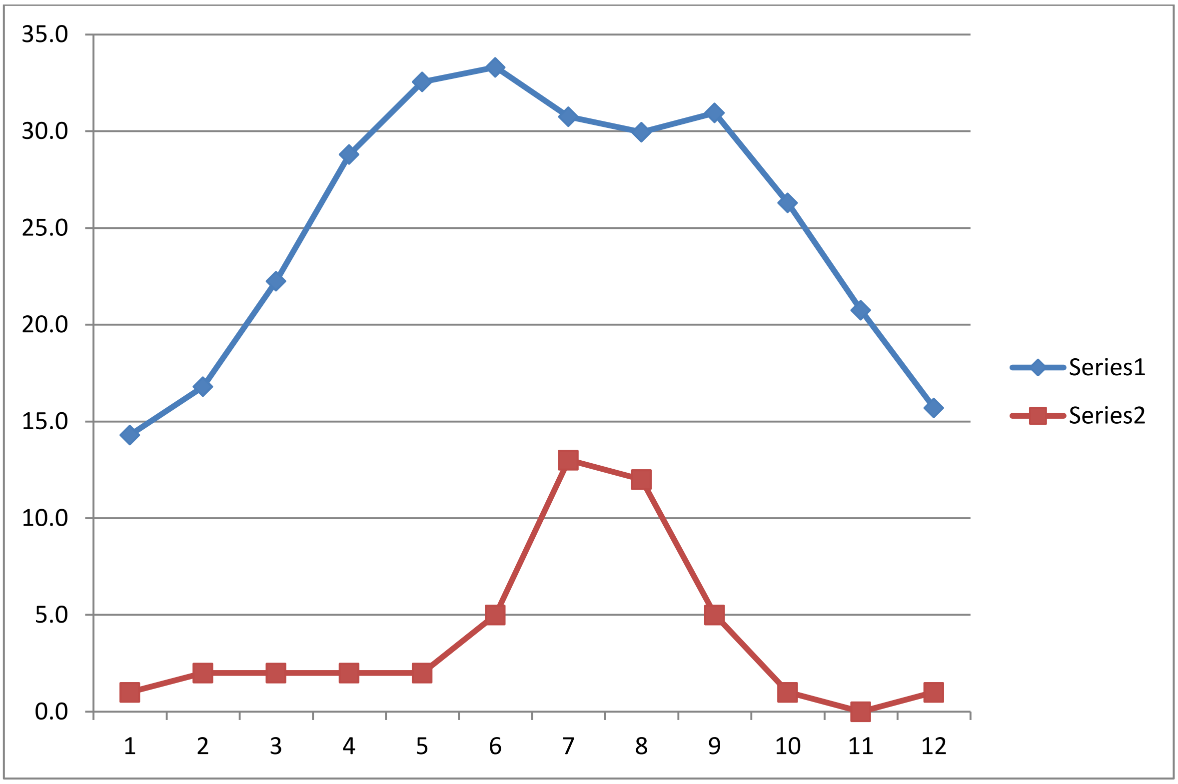
Procedure:-

1. We collect data of weather for a city from statistical department or from any other sources.
2. We represent the data graphically and find mean and coefficient of variance.
3. Interpretation of the data.

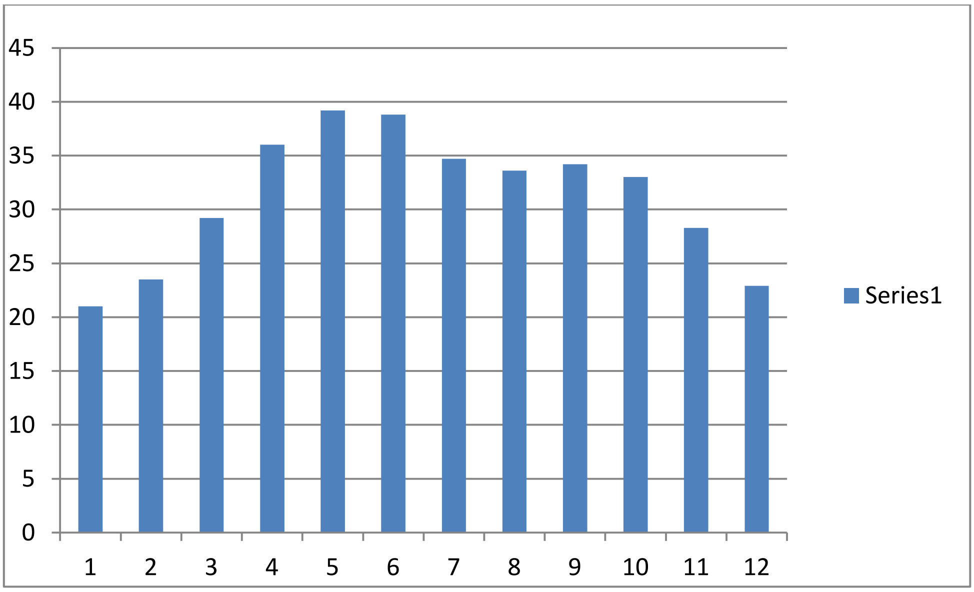
Demonstration:- We collect data of weather for New Delhi for the year 2017 as follows

Month	Average Temp. In Celsius	Warmest Temp	Coldest Temp.	Normal Precipitation
January	14.3	21	7.6	1
February	16.8	23.5	10.1	2
March	22.3	29.2	15.3	2
April	28.8	36	21.6	2
May	32.6	39.2	25.9	2
June	33.3	38.8	27.8	5
July	30.8	34.7	26.8	13
August	30.0	33.6	26.3	12
September	31.0	34.2	27.7	5
October	26.3	33	19.6	1
November	20.8	28.3	13.2	0
December	15.7	22.9	8.5	1

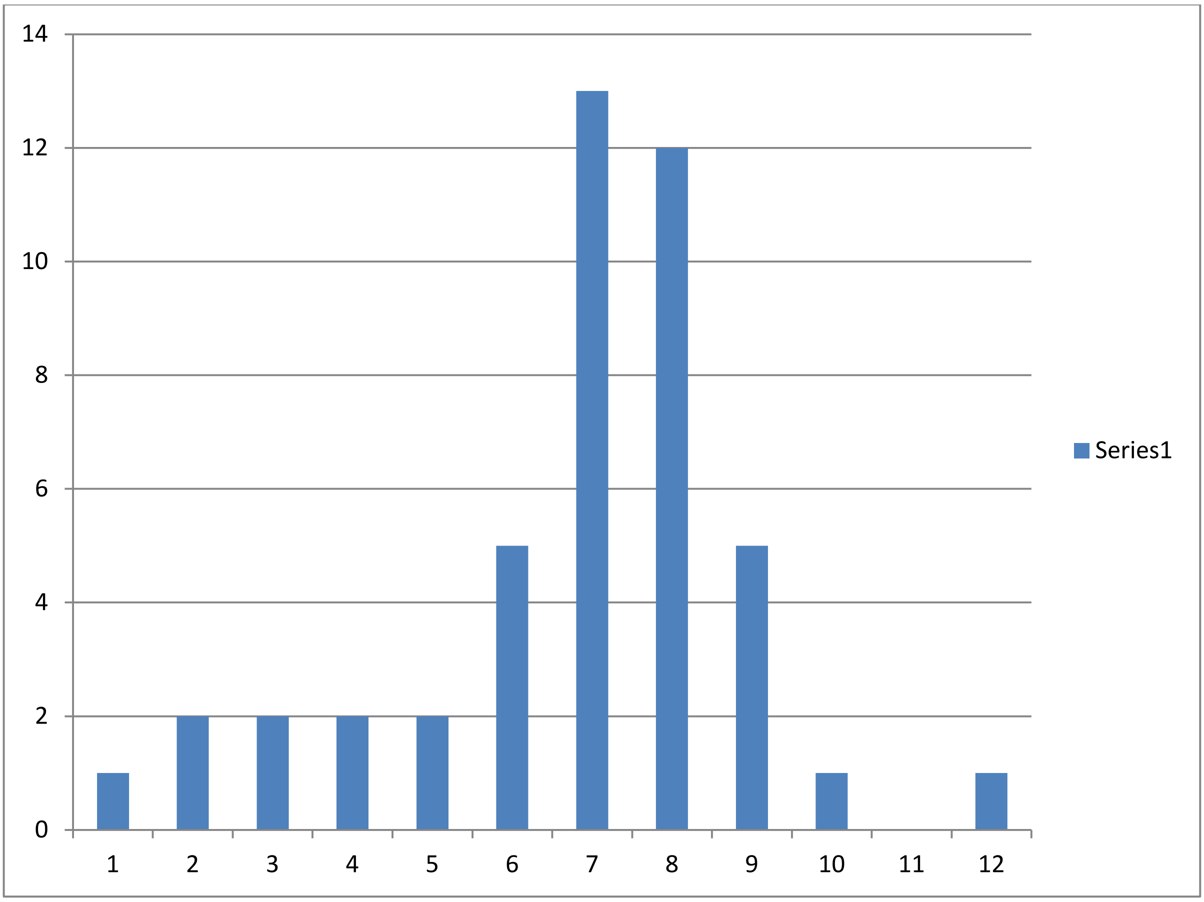
Graphical representation of data.



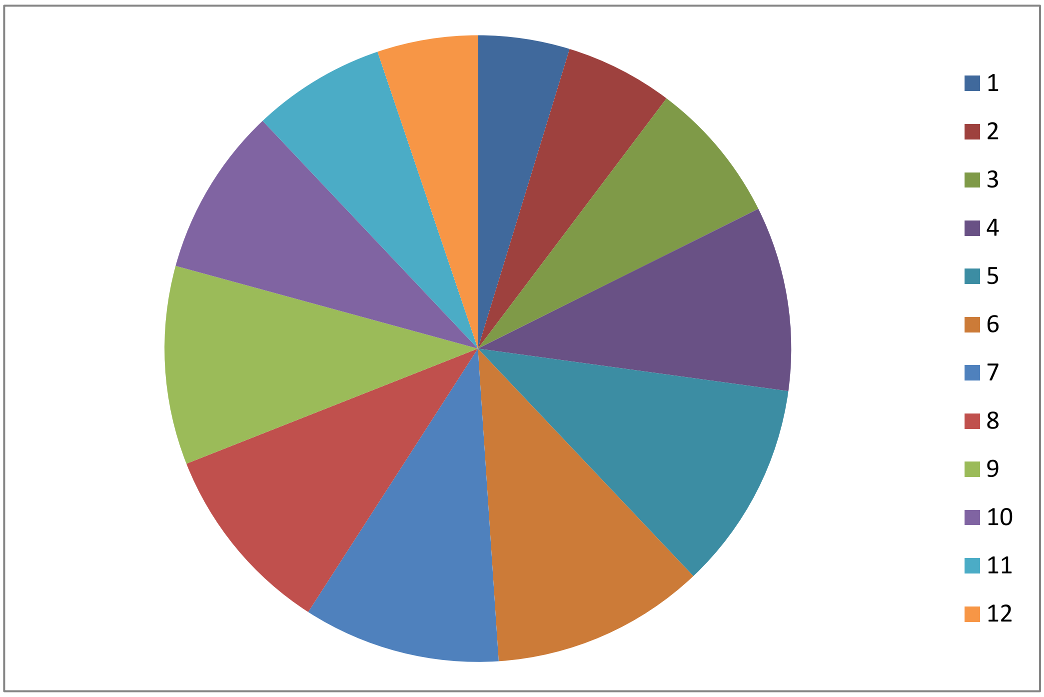
Relative scattering of temp and precipitation



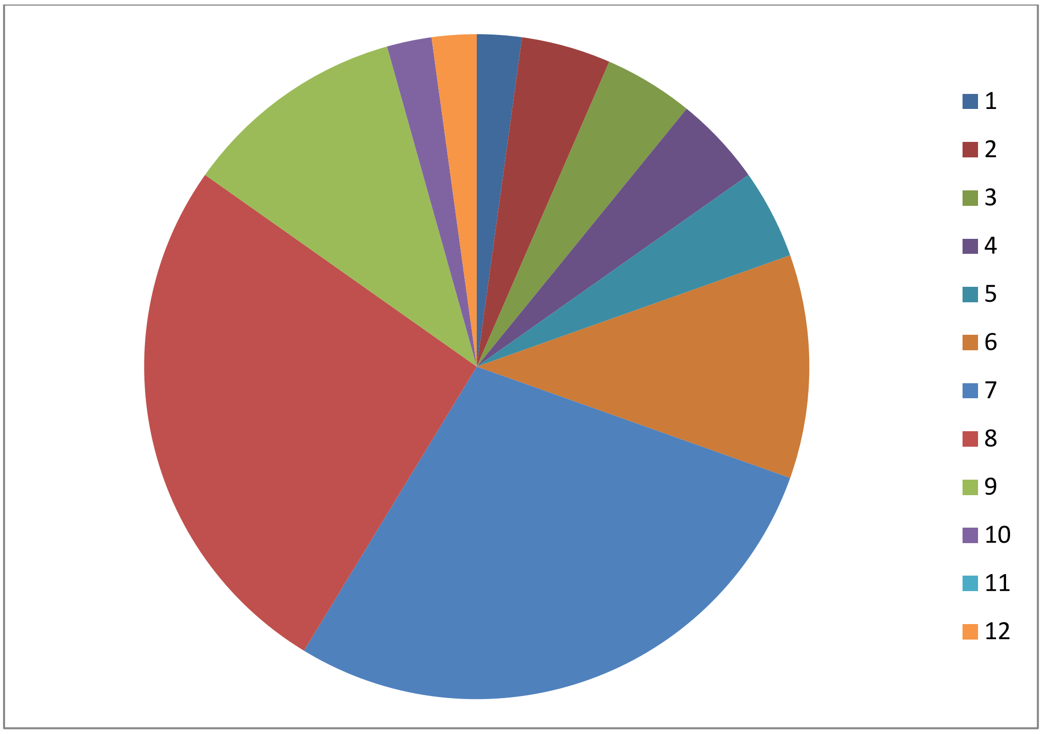
Months of 2017(Bar graph for temperature)



Months of 2017(Bar graph for precipitation)



Pie chart for Temperature



(Pie chart for Precipitation)

Here, 1:- January

2:- February

3:- March

4:- April

5:- May

6:- June

7:- July

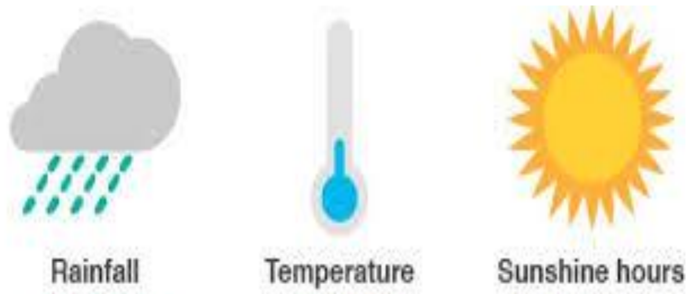
8:- August

9 :- September

10:-October

11:-November

12:-December



Calculation of Coefficient of variance

Months	Average Temp. In Celsius (x)	Average Precipitation. (y)	$(x-\bar{x})^2$	$(y-y)^2$
Jan.	14.3	1	118.81	8.01
Feb	16.8	2	70.56	3.35
March	22.3	2	8.70	3.35
April	28.8	2	12.96	3.35
May	32.6	2	54.02	3.35
June	33.3	5	65.61	1.37
July	30.8	13	30.80	84.09
August	30.0	12	22.56	66.75
Sept.	31.0	5	33.06	1.37
Oct.	26.3	1	1.21	8.01
Nov.	20.8	0	19.80	14.67
Dec.	15.7	1	90.25	8.01
Total	302.4	46.0	528.4	205.7

Mean of average Temperature=$\bar{x}=\frac{\sum x}{n}=\frac{302.4}{12}=25.2$

Mean of average Precipitation = $\bar{y}=\frac{\sum y}{n}=\frac{46}{12}=3.83$

Standard Deviation for Temperature (σ_x) =$\sqrt{\frac{\sum(x-\overline{x})^2}{n}}$=6.64

Standard Deviation for Precipitation($\sigma_{y)}$=$\sqrt{\frac{\sum(y-y^2}{n}}$= 4.14

C.V$_1$=Coefficient of Variance for Temperature = 26.33

C.V$_2$=Coefficient of Variance for Precipitation= $\frac{\sigma_y}{\bar{y}}X100=107.99.$

Interpretation of Data

From the analysis of the data it is clear that coefficient of variance of temp is less than that of precipitation, hence temperature is less scattered than precipitation,

CHAPTER:-16
TOPIC: PROBABILITY
Activity: 44

Aim: - To write the sample space , when a coin is tossed once, twice , thrice , four times, …, n times

Materials required: - 1) An unbiased coin, paper , pencil /pen , plastic circular disc marked with Head (H) and tail (T)

Procedure:-

1. Toss a coin once. It can have two outcomes : Head (H) and tail (T)
2. Make a tree diagram showing the two branches of tree: with H (head) on one branch and T (tail) on the other (as shown in the figure 1) .
3. Write its sample space.
4. Toss a coin twice it can have four outcomes (as shown in the figure 2).
5. Repeat the experiment tossing the coin three times , four times ,…, n times and write their sample spaces if possible – (as shown in the figure 3 &4)

Demonstration:-

1. If a coin is tossed once , the sample space S = { H, T }
   Number of elements in S = 2 = 2^1
2. When a coin is tossed twice ,the sample space S = {HH,HT, TH ,TT }
   Number of elements in S = 4 = 2^2
3. When a coin is tossed thrice ,the sample space
   S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
   Number of elements in S = 8 = 2^3
4. When a coin is tossed four times ,the sample space
   S = {HHHH,HHHT, HHTH , HHTT,HTHH,HTHT,HTTH,HTTT,THHH,THHT,THTH,THTT,TTHH,TTTH,TTHT,TTTT }
   Number of elements in S = 16 = 2^4

Figure 1

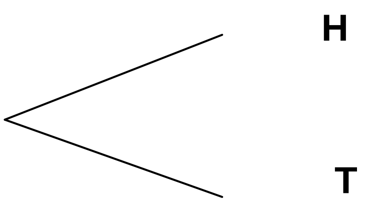


FIGURE 2

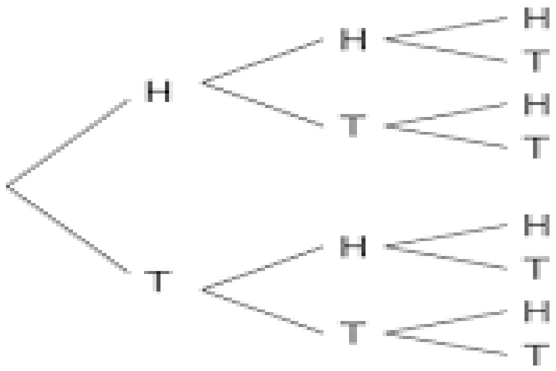
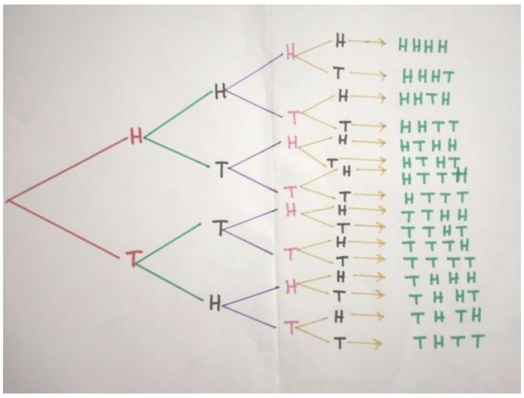


FIGURE: 3



Observation:-

Number of elements in sample space ,when a

1. Coin is tossed once = ------
2. Coin is tossed twice = ------
3. Coin is tossed thrice = ------
4. Coin is tossed four times = ------

Conclusion:- when a coin is tossed n times, then the number of elements of sample space is 2^n

Activity : 45

Aim: -To write the sample space, when a die is rolled once, twice, thrice , ….., n times .

Materials required: - An unbiased die, paper, pencil /pen, plastic circular disc marked with dots 1, 2,3,4,5 or 6

Procedure:-

1. Roll a die and note the number of dots on its top face, it will be 1,2,3,4,5 or 6(as shown in figure 1)
2. Write its sample space
3. Draw a tree diagram showing its six branches with numbers 1,2,3,4,5 or 6 (as shown in the figure 2) .
4. Roll a die twice. It can fall in any of the 36 ways (as shown in the figure 3)
5. Write its sample space.
6. Draw a tree diagram (as shown in the figure 4) .
7. Repeat the experiment ,rolling die thrice, four times, …n times and write their sample space if possible .

Demonstration:-

1. If a die is rolled once , The sample space is S = {1,2,3,4,5 ,6 }
   Number of elements in S = 6 = 6^1
2. When a die is rolled twice ,the sample space is S =
   {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
   (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
   (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}
   Number of elements in S = 36 = 6^2
3. When a a die is rolled thrice, number of elements in S = 216 = 6^3

Picture of faces of a Die

Figure 1

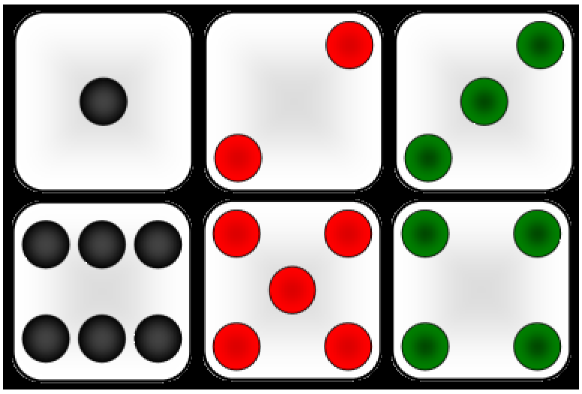


Figure 2

Tree diagram of sample space when 1 die rolled:

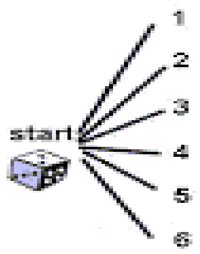


Figure 3

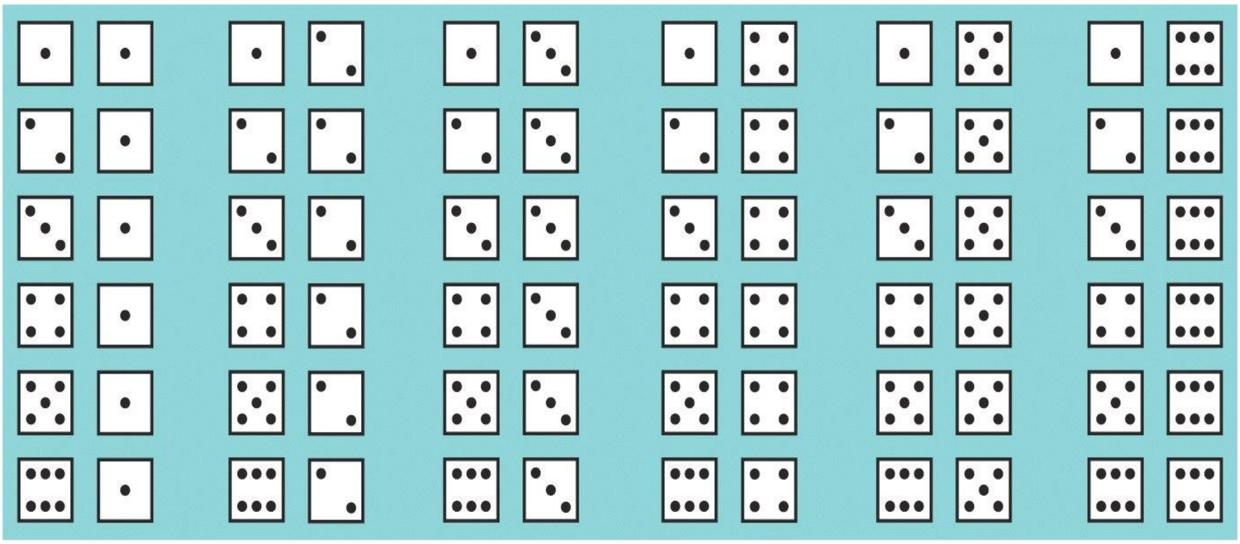
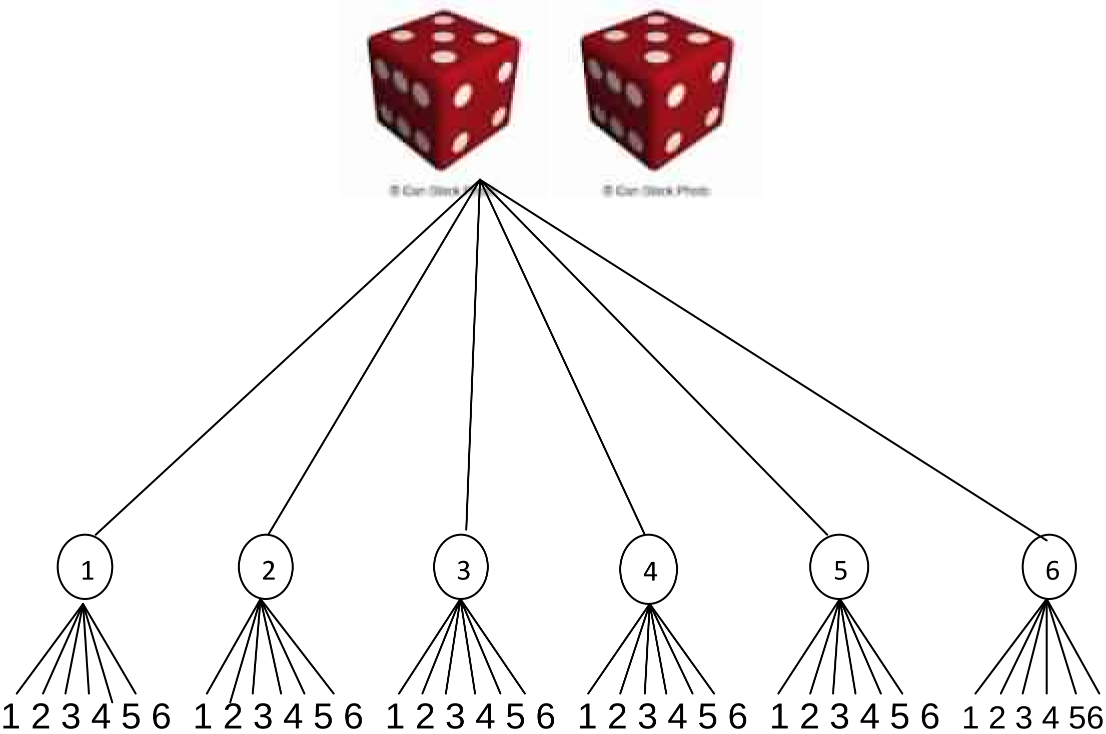


Figure 4

Tree diagram of sample space when 2 die rolled:



Observation:-

Number of element in sample space, when a

1. When a die is rolled once = ------
2. When a die is rolled twice = ------
3. When a die is rolled thrice = ------

…………………

…………………

Conclusion:- When a die is rolled n times, then the number of elements of sample space is 6^n