Navodaya Vidyalaya Samiti, (First Pre Board Exam 2018)

Marking Scheme (Set – I) Mathematics-XII

Section A

1. $|adj\ A| = 225 \Rightarrow |A| = \pm 15$

$|A'| = |A| = \pm 15$------------------ 1m

2. $\frac{d}{dx} sec(tan^{-1}x) = sec(tan^{-1}x)tan(tan^{-1}x)\frac{1}{1+x^2}$ -------------------- ½ m

$= \frac{x}{1+x^2} sec(tan^{-1}x)$ --------------------------------------- ½ m

3. $\vec{r} = (5\hat{\imath} - 4\hat{\jmath} + 6\hat{k}) + \mu(3\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$----------- 1m

OR

Direction ratios of the given line are 1, -7, 2 -------------- ½ m

Hence, direction ratios of any parallel line are 1,-7, 2 ---------- ½ m

4. $\int(e^{ax} + bx)dx = \frac{e^{4x}}{4} + 3x^2/2$

$\frac{e^{ax}}{a} + \frac{bx^2}{2} = \frac{e^{4x}}{4} + 3x^2/2$ -- ½ m

a = 4 and b = 3 --- ½ m

Section B

5. i) 'e' be the identity element in Q_0 then
a*e=e*a=a $\forall\ a \in Q_0 \Longrightarrow \frac{ae}{4} = a \Rightarrow e = 4$ ----------- 1 m

ii) Let 'a' be an invertible element in Q_0. Let 'b' be its inverse, then a*b=e=b*a
$\Rightarrow \frac{ab}{4} = 4 \Rightarrow b = \frac{16}{a} \in Q_0$ ----------- ----- 1 m

6. $f(x) = \begin{cases} x+2, if\ x < 1 \\ 0, if\ x = 1 \\ x-2, if\ x > 1 \end{cases}$

Discuss the continuity at $x < 1$ and $x > 1$ ---------------------- ½ m

Calculate RHL,LHL and f(1),and compare their values -------- 1 m

Then prove that they are equal ------------------------------ ½ m

OR

f(x) = $\begin{cases} 3ax - 1, x < -5 \\ 4, x \geq -5 \end{cases}$,taking LHL = RHL = f(-5) ----------------- 1m

using the condition , calculate value of a -------------------------- 1 m

7. If $e^y(x+1) = 1$, x + 1 = e^{-y} ------------------------------------ ½ m

differentiate with respect to y , $dx/dy = -e^{-y}$, -------------------------------1m

then $\frac{dy}{dx} = -e^{y}$ ---1/2 m

8. If $\begin{bmatrix} 2x-3y & z-w & 3 \\ 1 & x-4y & 3z+4w \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$

Taking 2x-3y =1,z-w = - 2, x-4y = 6, 3z + 4w = 29 -------------------- 1m

Solve equations to obtain value of x,y,z,w ----------------------------- 1m

9. $\int \frac{dx}{\sqrt{7-6x-x^2}} = \int \frac{dx}{\sqrt{16-(x+3)^2}}$ --------------------------------- 1m

Using the appropriate formula and integrate -------------------- 1m

10. A (1,1,1) B (1,2,3) and C (2,3,1).

Calculate $\overrightarrow{AB} \times \overrightarrow{AC}$ -- 1m

Find its magnitude -- ½ m

Using the formula of area of triangle ------------------------------- ½ m

OR

Write the correct formula for projection ----------------------- 1m

Correct answer ------------------------------------ 1m

11. $\frac{dy}{dx} = (1+x^2)(1+y^2)$

$dy/(1+y^2) = (1+x^2)\,dx$

integrate both sides to find solution ------------------------------ 2 m

12. $\int \frac{1}{x(x^n+1)}$ dx, let $x^n + 1 = t$, $x^{n-1}\,dx = dt/n$ -------------------- ½ m

$I = \frac{1}{n}\int \frac{dt}{t\,(t-1)}$ -- 1/2m

By using partial fraction form or another method , find solution ------------- 1m

OR

$\int (x^3 - 1)^{\frac{1}{3}}\, x^5\, dx$, taking $x^3 - 1 = t^3$, then $x^2\,dx = t^2\,dt$ ---------------------- 1 m

$I = \int (t^6 + t^3)\, dt$ -- ½ m

Integrate to find solution -------------------------------------- ½ m

Section C

13. $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2} \Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$ ------------------ ½m

$\Rightarrow 1-x = \sin(\frac{\pi}{2} + 2\sin^{-1}x) = \cos(2\sin^{-1}x)$ -------------------------------------1/2m

=>1-x=$\cos(2\alpha)$ where $\alpha = \sin^{-1} x$ ------------------------------------ 1/2m

=>1-x=1-2$\sin^2\alpha$=1-2x^2 --------------------------------------1m

$2x^2 - x = 0$

=>x(2x-1)=0 --- ½m

x= 0,1/2

Since x=1/2 does not satisfy the given equation

$\therefore$ x= 0 ---1m

14. The equation of family of circles is$(x-a)^2 + (y-b)^2 = 9$ ....(i) ------------------------- ½ m

=>$2(x-a) + 2(y-b)\frac{dy}{dx} = 0$ or $(x-a) = -(y-b)\frac{dy}{dx}$ ....(ii) ------------------------ 1 m

=>$1+(y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0 \Rightarrow y - b = -\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ ....(iii) ---------------------------- 1m

From (ii),$(x-a) = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}} dy/dx$ --- ½m

Putting in(i) to get

$$\left[\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2 \left(\frac{dy}{dx}\right)^2 + \left[\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2 = 9$$ -- ½m

Or$\left[\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2 \left[\left(\frac{dy}{dx}\right)^2 + 1\right] = 9$

=>$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = 9\left(\frac{d^2y}{dx^2}\right)^2$ -- ½ m

OR

$\frac{dy}{dx} + \frac{1}{x}.y = \frac{-1}{x(1+x^2)}$ 1

I.F = x 1

xy = $-\tan^{-1} x + C$ 1

Put x=1, y=0 . ...1/2

x.y = $-\tan^{-1} x + \frac{\pi}{4}$ *1/2*

15. Δ =1/abc $\begin{vmatrix} -abc & ab(b+c) & ac(b+c) \\ ab(a+c) & -abc & bc(a+c) \\ ac(a+b) & bc(b+a) & -abc \end{vmatrix}$ 1/2m

Δ =abc/abc $\begin{vmatrix} -bc & a(b+c) & a(b+c) \\ b(a+c) & -ac & b(a+c) \\ c(a+b) & c(b+a) & -ab \end{vmatrix}$ ----------------- 1/2m

By using $R_{1\rightarrow}\ R_1\ + R_2 + R_3$, taking common

= (ab+bc+ca). $\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & ab+bc \\ ac+bc & ca+cb & -ab \end{vmatrix}$ ………… 1

By using $C_2 \rightarrow C_2 - C_1$ *and* $C_3 \rightarrow C_3 - C_1$

= (ab+bc+ca). $\begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -(ab+bc+ca) & 0 \\ ac+bc & 0 & -(ab+bc+ca) \end{vmatrix}$ ……………… 1

= $(ab+bc+ca)^2$ ……………………. 1

16. Write correct formula for distance between two skew lines ------------------- 1m
Write values of components --1m
Correct calculation ------------------- 2m

17. Let $\vec{c} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, $\vec{a} \times \vec{c} = \overrightarrow{b} \Rightarrow \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{\jmath} - \hat{k}$ ---------- 1m

On simplifying, we get y=z, x=1+z, x-y=1-------- 1m
$\vec{a}.\vec{c} = 3 \Rightarrow x + y + z = 3$----------------- ½ m
On solving them , we get $x = \frac{5}{3}$ $y = \frac{2}{3}$ $z = \frac{2}{3}$--- 1m
$\therefore\ \vec{c} = \frac{5}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}$------- ½ m

18. I = $\int_0^{\pi} \frac{x\, sinx dx}{1+cos^2 x}$ ------------------ ½ m

I = $\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)dx}{1+cos^2(\pi-x)}$ --------------- ½ m

2I = $\pi \int_0^{\pi} \frac{sinx dx}{1+cos^2 x}$ --------------------- 1m

= $\pi \int_1^{-1} \frac{-dt}{1+t^2}$ *where* $t = cosx$ & $dt = -sinx\, dx$ ------------- 1m

$\therefore$ I = $\frac{\pi^2}{4}$ --------------------- 1m

19. Consider $\int \frac{x^2+4}{x^4+x^2+16} dx = \int \frac{1+\frac{4}{x^2}}{x^2+1+\frac{16}{x^2}} dx$ ------------- 1 m

=$\int \frac{dt}{t^2+3^2}$ *where* $t = x - \frac{4}{x}$ --------- 1 m

=$\frac{1}{3}\tan^{-1}\frac{t}{3}$ + c ---------------- 1 m

=$\frac{1}{3}\tan^{-1}\frac{x^2-4}{3x}$ + c --------------- 1 m

OR

$\int \frac{x^2}{(xsinx+cosx)^2} dx = \int \frac{(xsecx).(xcosx)}{(xsinx+cosx)^2} dx$ --1/2 m

Here u= x.sec x , $v = \frac{x\cos x}{(x\, sinx + cosx)^2}$,------------- ½ m
Using integration by parts ---------------------------1.5 m

Solve the integral to get answer -------------------- 1.5m

20. f '(x)=cos(x) + sin(x)

f ' (x)=0 → tan(x) = -1 (1)

x=3π/4 and 7π/4

Intervals are [0,3π/4),(3π/4,7π/4) , (7π/4,2π] (1)

On [0,3π/4), f ' (x) is positive, so f(x) is increasing

On (3π/4,7π/4), f '(x) is negative, so f(x) is decreasing

On (7π/4,2π], f ' (x)is positive, so f(x) is increasing (2)

21. Let y= $x^{xcosx} + \frac{x^2+1}{x^2-1}$; let y= u+v then
$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ --- ---- ½ m
$\frac{du}{dx} = x^{xcosx}(cosx - xlogx\; sinx + cosx\; logx)$----- 2m
$\frac{dv}{dx} = \frac{-4x}{(x^2-1)^2}$ -------- 1m
$\frac{dy}{dx} = x^{xcosx}(cosx - xlogx\; sinx + cosx\; logx)\frac{-4x}{(x^2-1)^2}$ ---------- ½ m

22. R is reflexive ……….1m

R is symmetric ………..1m

R is symmetric ……….1m

Hence R is an Equivalence relation ………. ½ m

[1,5,9] ……………………….1/2

OR

To show f is one-one . ------------------------------------ 3/2 m
To show f is onto . -------------------------------------- 3/2 m
To find f^{-1} . --- 1m
23. Let X denote the random variable ,'no.of green balls'

X	0	1	2	3 --------------- 1m
P(x)	${}^5C_3/{}^9C_3$ =5/42	${}^5C_2.{}^4C_1/{}^9C_3$ 10/21	${}^5C_1.{}^4C_2/{}^9C_3$ =5/14	${}^4C_3/{}^9C_3$ ------------------ 2 m =1/21 ----------- 1 m

Section D

24. $|A|$ = -3 (≠0) , $A^{-1} = \frac{1}{|A|}.adjA$ …………………………………………….. 1

The cofactor are $A_{11} = -1, A_{12} = -4\, , A_{13} = 1$

$A_{21} = 5, A_{22} = 23. A_{23} = -11$

$A_{31} = 3, A_{32} = 12, \; A_{33} = -6$ 2

$\therefore A^{-1} = -\frac{1}{3}\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & 11 & -6 \end{bmatrix}$ *1/2*

$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$ 1

X = $A^{-1}.B$

$\therefore \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & 11 & -6 \end{bmatrix} . \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -6 \\ -27 \\ 14 \end{bmatrix}$ *1*

x = -6 , y = -27 , z = 14 .... ½

OR

A= $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

AB= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ --------------------------- 2m

$\Rightarrow AB = I_3 \Rightarrow A^{-1} = B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ ----------------- 1m

The given system of equation can be written in matrix form as AX=C

Where A= $\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ B= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and C= $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$-------------- 1 m

AX=C$\Rightarrow X = A^{-1} C \Rightarrow X = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$----------- 1m

X= $\begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix} \Rightarrow x = 0, y = 5 \; and \; z = 3$------------------ 1m

25. Let E_1, E_2 and E_3 be the event to insure cyclist, scooter driver and motor bike driver respectively .P(E_1)=$\frac{2000}{12000} = \frac{1}{6}$, $P(E_2)$=$\frac{4000}{12000} = \frac{1}{3}$ and P(E_3) =$\frac{6000}{12000} = \frac{1}{2}$------ 1½ m

Let E be the event of an accident

$P\left(E/_{E_1}\right) = 0.01 = \frac{1}{100}$, $P\left(E/_{E_2}\right) = \frac{3}{100}$ & $P\left(E/_{E_3}\right) = \frac{15}{100}$---- 1 ½ m

For writing Baye's theorem formula-------------------------------- 1 m

$\therefore Req.probability = P\left(E_2/_{E}\right) = \frac{3}{26}$--------------------- 2 m

26. For finding correct shaded region --------------- 1m

For finding point of intersection, $(2, 2\sqrt{3})$ and $(2, -2\sqrt{3})$ — — — — --- 1m

Req. Area = $\pi(4)^2 - 2\left[\int_0^2 y\,dx(parabola) + \int_2^4 y\,dx\;(circle)\right]$----- 1m

$=16\,\pi - 2\left[\sqrt{6}\int_0^2 \sqrt{x}\, dx + \int_2^4 \sqrt{16-x^2}\;\; dx\right]$---- ½ m

For applying correct formula----------------- ½ m

We get,

$=16\pi - \left[\frac{16\sqrt{3}}{3} + (\frac{16\pi}{3} - 4\sqrt{3}\right]$------- 1m

Req. area= $\left[\frac{32\pi}{3} - \frac{4\sqrt{3}}{3}\right]$ sq. units.------- 1m

OR

Complete draw of diagram .... $1\frac{1}{2}$

Required Area = $\int_0^2 x^2 dx + \int_2^3 (x+2)dx$ 2

$= \left[\frac{x^3}{3}\right]_0^2 + \left[\frac{(x+2)^2}{2}\right]_2^3$ ....$1\frac{1}{2}$

$= \frac{8}{3} + \frac{25}{2}$ -8 $=\frac{43}{6}$ $sq.unit$ 1

27. Let x and y be the units of Food A and B respectively , then LPP is

Maximize z = 4x+3y .... ½

Subject to constraints

$200x + 100y \geq 4000$ or $2x + y \geq 40, \;\; x + 2y \geq 50$

40x + 40y ≥ 1400 or x+y ≥ 35, x ≥ 0 , y ≥ 0 .... $1\frac{1}{2}$

Correct graph 2

The corner of feasible region are A(50,0) , B(20,15), C(5,30), D(0,40) 1

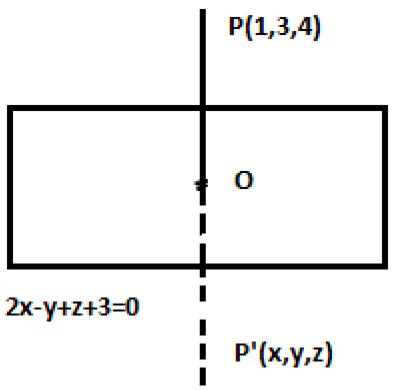
$Z_A = 200, Z_B = 125, Z_C = 120\,. \quad \therefore Z = 110$ $is\ minimum$

$\therefore$ 5 $units\ of\ Food\ \ a\ and$ 30 $units\ of\ Food\ B\ will\ give\ the\ minimum\ cost\ i.e\ Rs.$110

.... 1

28.Let O be foot of perpendicular from a point P & P' be the image of P .

The equation of line through P and O is $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1}$ *1*



The co-ordinate of O for some λ are $(2\lambda+1, -\lambda+3, \lambda+4)$ .... ½

Since O be on the plane , $\therefore 2(2\lambda+1) - 1(-\lambda+3) + 1(\lambda+4) + 3 = 0$ *1*

Solving the equation , $\lambda = -1$ .... ½

Co-ordinate of the point O be (-1,4,3) .... ½

Perpendicular distance PO = $\sqrt{6}$ units 1/2

Since O is mid-point of PP'

$\therefore \frac{x+1}{2} = -1, \frac{y+3}{2} = 4, \frac{z+4}{2} = 3, \Longrightarrow x = -3, y = 5, z = 2$ *1*

$\therefore$ Image of P is $(-3,5,2)$ 1

29. Let x be the side of square and r be the radius of a circle

$4x+2\pi r = k$ (say)---------------- 1m

Let A= Area of square+ Area of circle

$\therefore A = \frac{(k-2\pi r)^2}{16} + \pi r^2$----------------- 1m

$\frac{dA}{dr} = 2\pi r - \frac{\pi(k-2\pi r)}{4}$------------------ 1m

For minimum, $\frac{dA}{dr} = 0 \Rightarrow r = \frac{k}{2(4+\pi)}$------------------- 1m

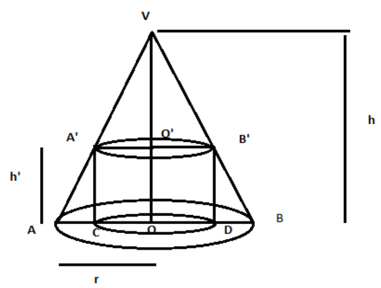
At $r = \frac{k}{2(4+\pi)}$, $\frac{d^2A}{dr^2} = 2\pi + \frac{\pi^2}{2} > 0$---------------------- 1m

Combined area is minimum at $r = \frac{k}{2(4+\pi)}$.

$$\therefore x == \frac{k - 2\pi\left[\frac{k}{2(4+\pi)}\right]}{4} = 2r$$

$\therefore$ *side of square = Diameter of the circle*------------- 1m

OR



. Let OC = x, $\Delta VOB \sim \Delta B'DB$ 1m

$\therefore \frac{VO}{B'D} = \frac{OB}{DB} \Longrightarrow B'D = \frac{h(r-x)}{r} = h'$

Let S = CSA of cylinder 1m

S = $2\pi x h' = 2\pi x\left[\frac{h(r-x)}{r}\right] = \frac{2\pi h}{r}[rx - x^2]$ *2m*

$\therefore \frac{dS}{dx} = \frac{2\pi h}{r}(r-2x), \frac{d^2S}{dx^2} = \frac{-4\pi h}{r} < 0 \Longrightarrow S \text{ is Maximum}$ 1m

$\frac{dS}{dx} = 0 \Longrightarrow r = 2x \Longrightarrow x = \frac{r}{2}$ *1m*