GANITA PRAKASH | GRADE 6 | CHAPTER 1

Patterns in Mathematics

The search for patterns — and the explanations for why they exist.

- Number Sequences
- Shape Sequences
- Patterns & Explanations

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Based on Ganita Prakash, Grade 6 (NCERT)

What is Mathematics?

An art and a science of finding patterns

The Search for Patterns

Mathematics is, in large part, the search for patterns — and for the explanations as to why those patterns exist. Patterns exist all around us: in nature, homes, schools, and the motion of the sun, moon, and stars.

Art + Science

Finding patterns is creative (an art). Explaining why they happen is rigorous (a science). Mathematicians enjoy both the discovery and the proof — that's what makes mathematics a fun, creative endeavour.

Why Explanations Matter

It is important to keep in mind that mathematics aims not just to find out what patterns exist, but also the explanations for why they exist. Such explanations can then be used in applications well beyond the context in which they were discovered — e.g., patterns in planetary motion led to the theory of gravitation (rockets to the Moon and Mars), and patterns in genomes help diagnose and cure diseases.

Patterns All Around Us

From everyday life to propelling humanity forward

1

Everyday Life

Shopping, cooking, throwing a ball, playing games, weather, and technology all involve recognising patterns.

2

Science

Patterns in the motion of stars & planets led to the theory of gravitation — enabling satellites and space travel.

3

Medicine

Understanding patterns in genomes has helped in diagnosing and curing diseases.

4

Technology

Building bridges, houses, TVs, phones, computers, cars, planes, calendars, and clocks all rely on mathematical patterns.

1.2 Patterns in Numbers

Number Theory: the branch of mathematics that studies patterns in whole numbers

Sequence	Example	Rule
All 1's	1, 1, 1, 1, 1, ...	Every term is 1
Counting numbers	1, 2, 3, 4, 5, ...	Add 1 each time
Odd numbers	1, 3, 5, 7, 9, ...	Add 2, start at 1
Even numbers	2, 4, 6, 8, 10, ...	Add 2, start at 2
Triangular numbers	1, 3, 6, 10, 15, ...	Add next counting number
Square numbers	1, 4, 9, 16, 25, ...	n × n
Cubes	1, 8, 27, 64, 125, ...	n × n × n
Virahānka numbers	1, 2, 3, 5, 8, 13, ...	Sum of previous two terms
Powers of 2	1, 2, 4, 8, 16, ...	× 2 each time
Powers of 3	1, 3, 9, 27, 81, ...	× 3 each time

Visualising the Basics

All 1's, Counting, Odd, and Even numbers as dot pictures

Triangular Numbers

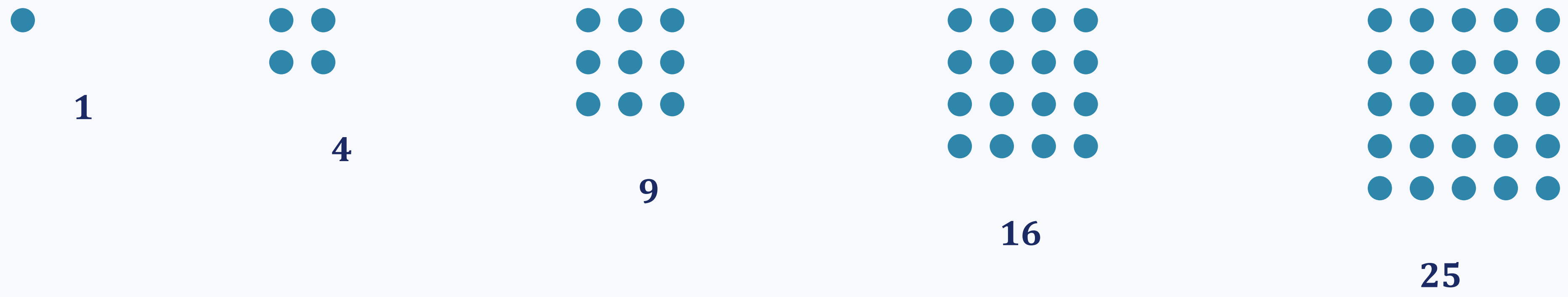
1, 3, 6, 10, 15, ... — formed by stacking rows of dots

Why 'Triangular'?

Each term adds the next counting number to the previous one: 1, 1+2=3, 1+2+3=6, 1+2+3+4=10, 1+2+3+4+5=15 ... The dots for each term can be arranged perfectly into a triangle — hence the name. Fun fact: 36 is both a triangular number and a square number!

Square Numbers

1, 4, 9, 16, 25, ... — dots arranged in a perfect n × n grid

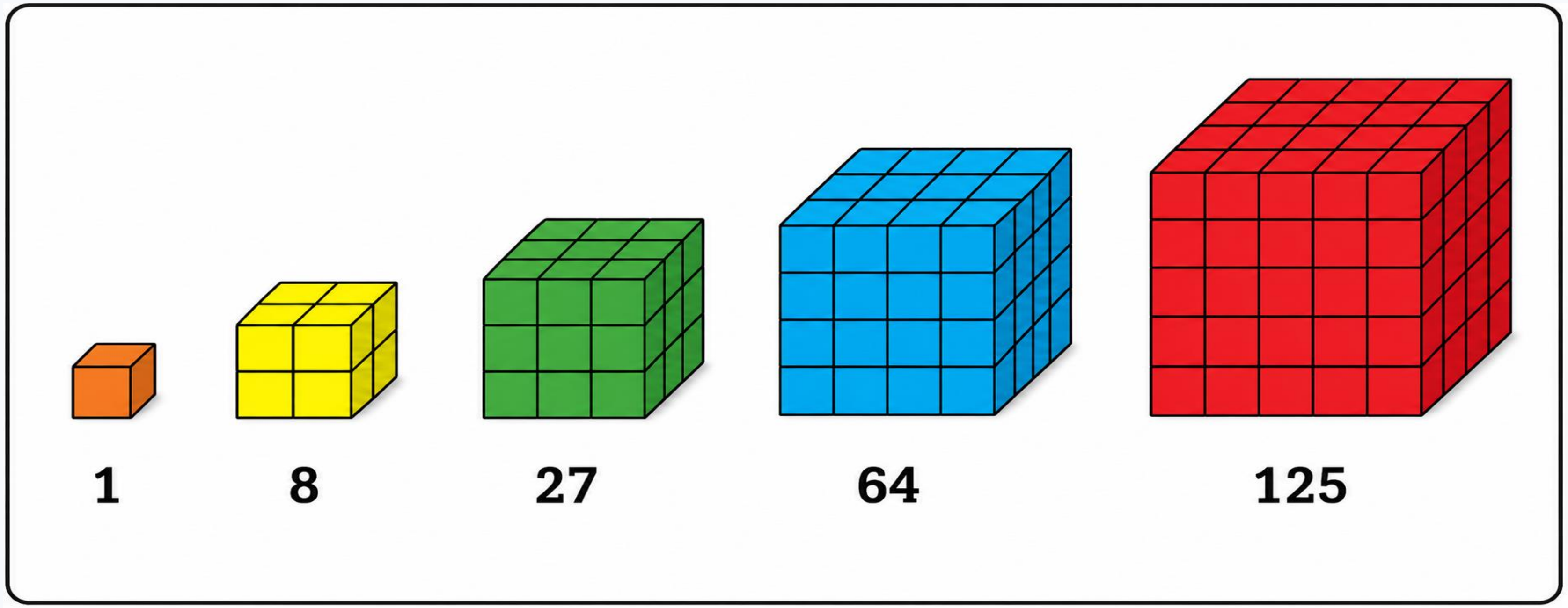


Why 'Square'?

Each term is $n \times n$ (n^2): 1×1=1, 2×2=4, 3×3=9, 4×4=16, 5×5=25 ... The dots for each term can be arranged perfectly into a square grid with equal rows and columns — hence the name 'square numbers'.

Cube Numbers

1, 8, 27, 64, 125, ... — units arranged in a perfect 3-D cube (n × n × n)



Why 'Cubes'?

Each term is $n \times n \times n$ (n^3): $1^3=1$, $2^3=8$, $3^3=27$, $4^3=64$, $5^3=125$ ... Unit blocks can be stacked to form a perfect 3-D cube — hence the name.

Virahānka Numbers

1, 2, 3, 5, 8, 13, 21, ... — India's own "Fibonacci" sequence

1	2	3	5	8	13	21
		=1+2	=2+3	=3+5	=5+8	=8+13

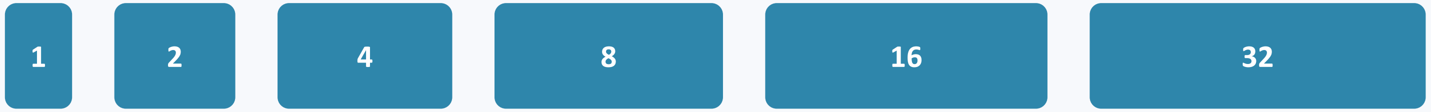
The Rule

Each number (from the third term onward) is the sum of the two numbers before it: 2+3=5, 3+5=8, 5+8=13, 8+13=21, and so on. This sequence is named after the Indian mathematician Virahānka, who described it centuries before Fibonacci. It appears in nature — e.g., the spiral patterns of shells, flowers, and pinecones.

Powers of 2 and Powers of 3

Doubling and tripling patterns

Powers of 2: 1, 2, 4, 8, 16, 32, 64, ...



Powers of 3: 1, 3, 9, 27, 81, 243, ...

The Rule

Powers of 2: each term is 2 × the previous term (2, 2×2=4, 2×2×2=8, ...). Powers of 3: each term is 3 × the previous term (3, 3×3=9, 3×3×3=27, ...). These can be visualised using dots, lines, squares, and cubes of increasing dimension (1D → 2D → 3D → beyond)!

1.4 A Beautiful Relation

Adding up odd numbers always gives a square number!

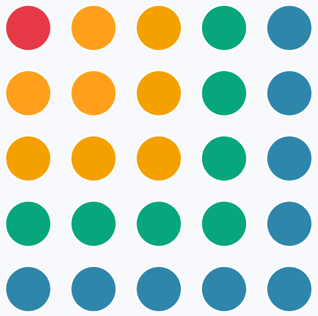
$1 = 1$

$1 + 3 = 4$

$1 + 3 + 5 = 9$

$1 + 3 + 5 + 7 = 16$

$\mathbf{1 + 3 + 5 + 7 + 9 = 25}$



$5^2 = 25$

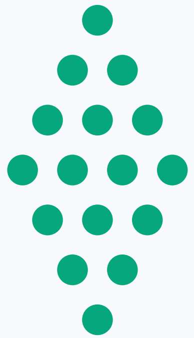
Why does this happen?

A square grid of dots can be split into L-shaped colour bands of 1, 3, 5, 7, 9 ... dots (see picture). Since each new band adds the next odd number and together they always fill a perfect square, the sum of the first n odd numbers always equals n^2 — forever!

More Beautiful Relations

Adding counting numbers up-and-down, and triangular pairs

Up-and-Down Sums



1+2+3+4+3+2+1 = 16 = 4^2

Consecutive Triangular Numbers

6 10

6 + 10 = 16 = 4^2

The Patterns

Adding counting numbers up to n and back down to 1 always gives n^2 (visualised as a diamond of dots). Adding two consecutive triangular numbers also always gives a square number, because two triangles of dots fit together perfectly to form a square. Try: 1+2+3+...+99+100+99+...+2+1 = 100^2 = 10,000!

Hexagonal Numbers → Cubes

Triangular numbers × 6 + 1 gives hexagonal numbers

1	3	6	10	15
×6+1	×6+1	×6+1	×6+1	×6+1
7	19	37	61	91

Triangular numbers (orange) → Hexagonal numbers (purple): 1, 7, 19, 37, 61, 91, ...

Adding Hexagonal Numbers Gives Cubes!

1 = 1, 1+7 = 8, 1+7+19 = 27, 1+7+19+37 = 64 ... Adding up hexagonal numbers in sequence always produces the cube numbers (1, 8, 27, 64, ...). This can be explained by imagining each hexagonal 'layer' of dots as a cross-sectional slice used to build up a 3-D cube.

1.5 Patterns in Shapes

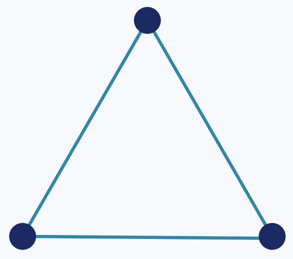
Regular Polygons — studied by Geometry

Complete Graphs

K2, K3, K4, K5, K6 — every point joined to every other point

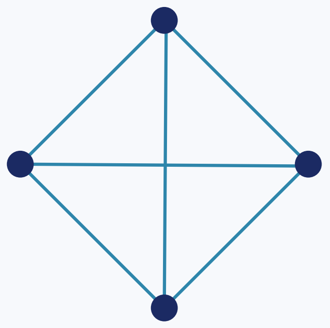
K2

1 lines



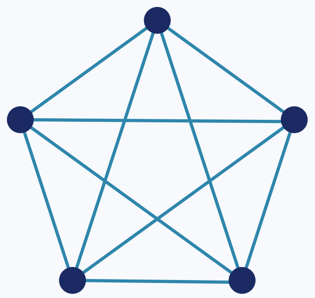
K3

3 lines



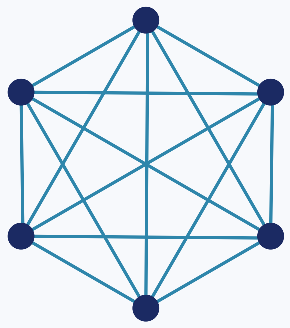
K4

6 lines



K5

10 lines



K6

15 lines

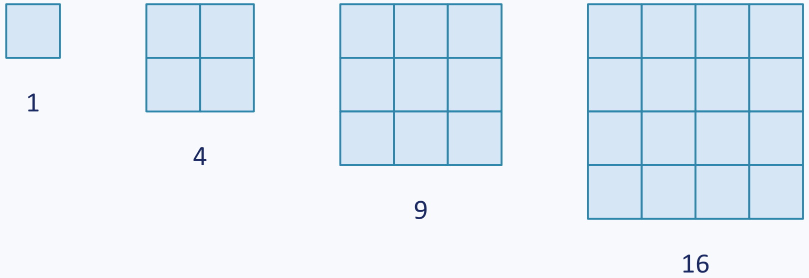
The Pattern

The number of lines in K2, K3, K4, K5, K6 is 1, 3, 6, 10, 15 — the Triangular Number sequence! This happens because every point connects to every other point exactly once, giving 1+2+3+...+(n-1) total connections.

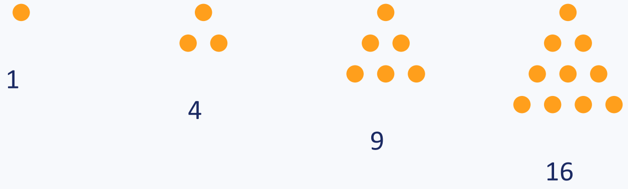
Stacked Shapes & the Koch Snowflake

More shape sequences with hidden number patterns

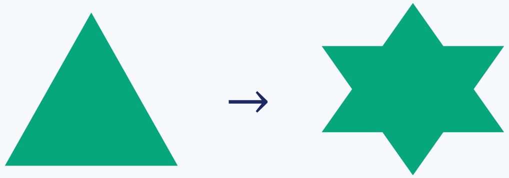
Stacked Squares



Stacked Triangles



Koch Snowflake



→ ... *48, 192, 768 segments*

Step 1: 3 segments

Step 2: 12 segments

The Patterns

Stacked Squares & Triangles: number of unit shapes = 1, 4, 9, 16, 25 ... (Square numbers). Koch Snowflake: each straight segment is replaced by a 'speed bump', so the number of segments follows 3, 12, 48, 192 ... (3 × Powers of 4).

1.6 Shapes Meet Numbers

How shape sequences connect to number sequences

Shape Sequence	What We Count	Related Number Sequence
Regular Polygons	Number of sides/corners	3, 4, 5, 6, 7, 8, 9, 10 (Counting numbers from 3)
Complete Graphs	Number of connecting lines	1, 3, 6, 10, 15 (Triangular numbers)
Stacked Squares	Number of unit squares	1, 4, 9, 16, 25 (Square numbers)
Stacked Triangles	Number of unit triangles	1, 4, 9, 16, 25 (Square numbers)
Koch Snowflake	Number of line segments	3, 12, 48, 192 (3 × Powers of 4)

Key Insight

In any closed shape, the number of sides always equals the number of corners — because each side connects two corners and each corner joins two sides, going all the way around.

Key Takeaways

1. Mathematics is the search for patterns AND the explanations for why they exist.
2. Number sequences: All 1's, Counting, Odd, Even, Triangular, Square, Cube, Virahānka, Powers of 2 & 3.
3. Sum of the first n odd numbers = n^2 — a beautiful, provable relation.
4. Pictures and diagrams help us visualise and explain WHY number patterns work.
5. Shape sequences — Regular Polygons, Complete Graphs, Stacked Squares/Triangles, Koch Snowflake — connect beautifully to number sequences.