GANITA PRAKASH — GRADE 6

Chapter 1: Patterns in Mathematics

Complete Study Material — Notes, MCQs, Short Answer, Long Answer & Homework Questions (with Explained Answers)

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A. Notes — Key Concepts

1. What is Mathematics?

Mathematics is, in large part, the search for patterns and the explanations for why those patterns exist. Patterns are found everywhere — in nature, shopping, cooking, games, weather, and technology.

Mathematics is both an art (creativity in discovering patterns) and a science (rigorous explanation of why patterns hold). Finding the explanation behind a pattern is as important as the pattern itself, because explanations can be applied far beyond the original context (e.g., patterns in planetary motion led to the theory of gravitation and space travel; patterns in genomes help diagnose diseases).

2. Patterns in Numbers

The branch of mathematics that studies patterns in whole numbers is called Number Theory. A Number Sequence is an ordered list of numbers following a rule.

Important Number Sequences (Table 1)

Sequence	Example	Rule
All 1's	1, 1, 1, 1, 1, ...	Every term is 1
Counting numbers	1, 2, 3, 4, 5, ...	Add 1 to previous term
Odd numbers	1, 3, 5, 7, 9, ...	Add 2 to previous term (starting at 1)
Even numbers	2, 4, 6, 8, 10, ...	Add 2 to previous term (starting at 2)
Triangular numbers	1, 3, 6, 10, 15, ...	Add the next counting number (1, +2, +3, +4...)
Square numbers	1, 4, 9, 16, 25, ...	$n \times n$, i.e., n^2
Cubes	1, 8, 27, 64, 125, ...	$n \times n \times n$, i.e., n^3
Virahānka (Fibonacci) numbers	1, 2, 3, 5, 8, 13, 21, ...	Each term = sum of previous two terms
Powers of 2	1, 2, 4, 8, 16, 32, ...	Each term = 2 × previous term
Powers of 3	1, 3, 9, 27, 81, 243, ...	Each term = 3 × previous term

Why the names?

- Triangular numbers: dots can be stacked to form a triangle (1 dot, then rows of 2, 3, 4 dots added below).
- Square numbers: dots can be arranged in a perfect square grid (n rows × n columns).
- Cubes: units can be arranged in a perfect 3-D cube ($n \times n \times n$).
- Interesting fact: 36 is both a triangular number AND a square number — the same 36 dots can be arranged as a triangle or as a 6×6 square.

3. Visualising Number Sequences

Pictures/diagrams help understand why patterns occur. For example, square numbers can be visualised as dots arranged in a square grid, and cubes as a 3-D stack of unit cubes.

4. Relations Among Number Sequences

(a) Sum of odd numbers = Square numbers

1 = 1; 1+3 = 4; 1+3+5 = 9; 1+3+5+7 = 16; 1+3+5+7+9 = 25; and so on. In general, the sum of the first n odd numbers = n^2. This can be shown by partitioning a square dot-grid into L-shaped bands of 1, 3, 5, 7 ... dots.

Sum of first 10 odd numbers = 10^2 = 100. Sum of first 100 odd numbers = 100^2 = 10,000.

(b) Adding counting numbers up-and-down = Square numbers

1 = 1; 1+2+1 = 4; 1+2+3+2+1 = 9; 1+2+3+4+3+2+1 = 16 ... In general, adding 1 to n and back down to 1 gives n^2. So, 1+2+3+...+99+100+99+...+2+1 = 100^2 = 10,000.

(c) Adding counting numbers (only going up) = Triangular numbers

1, 1+2, 1+2+3, 1+2+3+4, ... gives 1, 3, 6, 10, ... which is the triangular number sequence. This can be shown using half of a rectangular grid (a triangular half).

(d) Sum of consecutive triangular numbers = Square numbers

1+3 = 4, 3+6 = 9, 6+10 = 16, 10+15 = 25 — adding two consecutive triangular numbers always gives a square number. This is because two triangles fit together to form a square.

(e) Sum of powers of 2 (+1) = Next power of 2

1, 1+2=3, 1+2+4=7, 1+2+4+8=15, ... Adding 1 to each of these sums gives 2, 4, 8, 16, ... i.e., the powers of 2 themselves. In general, $1+2+4+...+2^n = 2^{(n+1)} - 1$.

(f) Triangular numbers × 6, + 1 = Hexagonal numbers

(1×6)+1=7, (3×6)+1=19, (6×6)+1=37, (10×6)+1=61 ... gives the hexagonal number sequence 1, 7, 19, 37, 61, ...

(g) Sum of hexagonal numbers = Cubes

1, 1+7=8, 1+7+19=27, 1+7+19+37=64 ... gives the cube number sequence 1, 8, 27, 64, ...

5. Patterns in Shapes

The branch of mathematics that studies patterns in shapes is called Geometry. Shapes may exist in 1D, 2D, 3D, or more dimensions.

Important Shape Sequences (Table 3)

Shape sequence	Description	Related number sequence
Regular Polygons	Triangle, Quadrilateral, Pentagon, Hexagon, Heptagon, Octagon, Nonagon, Decagon...	Number of sides/corners: 3, 4, 5, 6, 7, 8, 9, 10... (counting numbers from 3)
Complete Graphs (K2, K3, K4...)	All points joined to every other point by a line	Number of lines: 1, 3, 6, 10, 15... (Triangular numbers)
Stacked Squares	Squares built from smaller unit squares, growing each time	Number of unit squares: 1, 4, 9, 16, 25... (Square numbers)
Stacked Triangles	Triangles built from smaller unit triangles	Number of unit triangles: 1, 4, 9, 16, 25... (Square numbers, from up+down counting)
Koch Snowflake	Each straight segment is replaced by a 'speed bump' shape repeatedly	Number of segments: 3, 12, 48, 192... (3 × Powers of 4)

A regular polygon has all sides of equal length and all angles equal. The word 'regular' captures this symmetry.

6. Summary Points (Quick Revision)

- Mathematics = search for patterns + explanations for why they exist.
- Number sequences: All 1's, Counting, Odd, Even, Triangular, Squares, Cubes, Virahānka (Fibonacci), Powers of 2, Powers of 3.
- Sum of first n odd numbers = n^2 (a key beautiful relation).
- Pictures/diagrams help visualise and prove why number patterns work.
- Shape sequences: Regular Polygons, Complete Graphs, Stacked Squares, Stacked Triangles, Koch Snowflake.

- Shape sequences are often linked to number sequences (e.g., sides of regular polygons = counting numbers from 3; lines in complete graphs = triangular numbers).

B. Multiple Choice Questions (MCQs)

Each question carries 1 mark. Choose the correct option.

Q1. Mathematics is, in large part, the search for:

(A) Numbers only (B) Patterns and explanations for them (C) Shapes only (D) Calculations

Answer: (B) Patterns and explanations for them ***Explanation:*** *As stated in the chapter, mathematics aims not just to find patterns, but also to explain why they exist.*

Q2. The branch of mathematics that studies patterns in whole numbers is called:

(A) Geometry (B) Algebra (C) Number theory (D) Statistics

Answer: (C) Number theory ***Explanation:*** *The text explicitly states: 'The branch of Mathematics that studies patterns in whole numbers is called number theory.'*

Q3. Which of these is the sequence of triangular numbers?

(A) 1, 4, 9, 16, 25 (B) 1, 3, 6, 10, 15 (C) 1, 2, 4, 8, 16 (D) 1, 3, 5, 7, 9

Answer: (B) 1, 3, 6, 10, 15 ***Explanation:*** *Triangular numbers are formed by adding consecutive counting numbers: 1, 1+2=3, 1+2+3=6, 1+2+3+4=10, 1+2+3+4+5=15.*

Q4. The sum of the first 10 odd numbers is:

(A) 55 (B) 100 (C) 90 (D) 110

Answer: (B) 100 ***Explanation:*** *The sum of the first n odd numbers is n^2. For n = 10, sum = 10^2 = 100.*

Q5. 36 is special because it is:

(A) Only a square number (B) Only a triangular number (C) Both a square and a triangular number (D) A prime number

Answer: (C) Both a square and a triangular number ***Explanation:*** *36 dots can be arranged both as a 6×6 square and as a triangle (1+2+3+4+5+6+7+8=36), so it appears in both sequences.*

Q6. The next number in the Virahānka (Fibonacci) sequence 1, 2, 3, 5, 8, 13, 21, ... is:

(A) 26 (B) 34 (C) 33 (D) 30

Answer: (B) 34 ***Explanation:*** *Each term is the sum of the previous two terms: 13 + 21 = 34.*

Q7. A shape with 7 equal sides and 7 equal angles is called a:

(A) Hexagon (B) Octagon (C) Heptagon (D) Nonagon

Answer: (C) Heptagon ***Explanation:*** *Regular polygons are named by their number of sides: 3-triangle, 4-quadrilateral, 5-pentagon, 6-hexagon, 7-heptagon, 8-octagon, 9-nonagon, 10-decagon.*

Q8. In the sequence of Complete Graphs (K2, K3, K4, K5...), the number of lines follows the pattern:

(A) Square numbers (B) Triangular numbers (C) Powers of 2 (D) Odd numbers

Answer: (B) Triangular numbers ***Explanation:*** *The number of connecting lines in K2, K3, K4, K5... is 1, 3, 6, 10, 15..., which is the triangular number sequence.*

Q9. Adding 1 + 2 + 3 + 4 + 3 + 2 + 1 (up and down) gives:

(A) 16 (B) 25 (C) 12 (D) 20

Answer: (A) 16 ***Explanation:*** *Adding counting numbers up to n and back down always gives n^2. Here n = 4, so the sum = 4^2 = 16.*

Q10. The Koch Snowflake sequence of total line segments is:

(A) 3, 6, 9, 12, ... (B) 3, 12, 48, 192, ... (C) 1, 4, 9, 16, ... (D) 3, 9, 27, 81, ...

Answer: (B) 3, 12, 48, 192, ... ***Explanation:*** *Each segment is replaced by 4 smaller segments at every step, starting from 3 segments (a triangle), giving 3 × Powers of 4 = 3, 12, 48, 192, ...*

C. Short Answer Questions (2–3 marks each)

Q1. Why are the numbers 1, 4, 9, 16, 25, ... called square numbers?

Answer: They are called square numbers because that many dots can be arranged perfectly in a square grid — for example, 9 dots form a 3×3 square, and 16 dots form a 4×4 square. Mathematically, each term is $n \times n = n^2$ for n = 1, 2, 3, 4, 5, ...

Q2. What is the relationship between odd numbers and square numbers?

Answer: Adding up consecutive odd numbers starting from 1 always gives a square number: $1 = 1^2$, $1+3 = 2^2$, $1+3+5 = 3^2$, $1+3+5+7 = 4^2$, and so on. This can be shown pictorially by dividing a square dot-grid into L-shaped bands of 1, 3, 5, 7... dots.

Q3. How are triangular numbers formed? Give the first five terms.

Answer: Triangular numbers are formed by adding consecutive counting numbers: 1, 1+2=3, 1+2+3=6, 1+2+3+4=10, 1+2+3+4+5=15. The first five terms are 1, 3, 6, 10, 15. They are called triangular because that many dots can be arranged in a triangular shape.

Q4. What are Virahānka numbers? Write the rule for forming them.

Answer: Virahānka numbers (also called Fibonacci numbers) are 1, 2, 3, 5, 8, 13, 21, ... The rule is that every number (from the third term onward) is the sum of the two numbers before it: 2+3=5, 3+5=8, 5+8=13, and so on.

Q5. Explain why the number of sides and number of corners is the same for any regular polygon sequence.

Answer: In any closed shape (polygon), every side connects two corners, and every corner joins exactly two sides — the sides and corners always occur in matching pairs going around the closed figure. Hence, for any polygon, the number of sides always equals the number of corners (vertices).

Q6. What number sequence do you get by counting the number of lines in the Complete Graph sequence K2, K3, K4, K5, ...? Explain briefly.

Answer: You get the triangular number sequence: 1, 3, 6, 10, 15, ... This happens because in a complete graph with n points, every point is joined to every other point exactly once, and the total number of such connections works out to 1+2+3+...+(n-1), which is a triangular number.

D. Long Answer Questions (4–5 marks each)

Q1. Explain, with a picture-based reasoning, why adding up the first n odd numbers always gives a perfect square (n^2). Illustrate using the first 4 odd numbers.

Answer:

- A square number can be represented as dots arranged in an $n \times n$ grid.
- If we look at a square grid, we can divide (partition) it into L-shaped bands starting from one corner: the first band has 1 dot, the next L-shaped band around it has 3 dots, the next has 5 dots, then 7 dots, and so on.
- Since each new band adds an odd number of dots (1, 3, 5, 7, ...), and all these bands together fill up exactly the $n \times n$ square, the sum of the first n odd numbers must equal n^2.
- Illustration with first 4 odd numbers: $1 + 3 + 5 + 7 = 16 = 4^2$. Here, the 1st band has 1 dot, forming a 1×1 square; adding the next band of 3 dots extends it to a 2×2 square (4 dots); adding 5 more dots extends it to a 3×3 square (9 dots); adding 7 more dots extends it to a 4×4 square (16 dots).
- This shows that the pattern is not a coincidence — the geometric arrangement explains exactly why it happens, and this reasoning extends to any value of n, so the pattern continues forever.

Q2. Describe the shape sequences given in the chapter (Regular Polygons, Complete Graphs, Stacked Squares, Stacked Triangles, Koch Snowflake) and state the number sequence associated with each.

Answer:

- Regular Polygons: Triangle, Quadrilateral, Pentagon, Hexagon, Heptagon, Octagon, Nonagon, Decagon — each has one more side than the previous shape. The associated number sequence (number of sides/corners) is the counting numbers starting from 3: 3, 4, 5, 6, 7, 8, 9, 10.
- Complete Graphs (K2, K3, K4, K5, K6): points joined to every other point by straight lines. The number of lines follows the triangular number sequence: 1, 3, 6, 10, 15.

- Stacked Squares: squares built from progressively more unit squares. The number of unit squares follows the square number sequence: 1, 4, 9, 16, 25.
- Stacked Triangles: triangles built from progressively more unit triangles, arranged in rows. The number of unit triangles also follows the square number sequence: 1, 4, 9, 16, 25 (obtained by adding row counts up and down).
- Koch Snowflake: each straight segment is replaced repeatedly by a 'speed bump' shape (4 smaller segments in place of 1). The total number of line segments follows 3, 12, 48, 192, ..., i.e., 3 × Powers of 4.

Q3. Using pictures/dot diagrams, show two different ways of arriving at the square number sequence from other sequences discussed in the chapter.

Answer:

- Way 1 — Sum of odd numbers: 1 = 1, 1+3 = 4, 1+3+5 = 9, 1+3+5+7 = 16, 1+3+5+7+9 = 25. This can be visualised as L-shaped bands of dots being added around a growing square (as explained above).
- Way 2 — Adding counting numbers up and down: 1 = 1, 1+2+1 = 4, 1+2+3+2+1 = 9, 1+2+3+4+3+2+1 = 16, 1+2+3+4+5+4+3+2+1 = 25. This can be visualised as a diamond (rotated square) shape of dots, where each row has 1, 2, 3, ..., n, ..., 3, 2, 1 dots, and the total number of dots forms a square when rearranged.
- Both methods, though based on very different-looking sums, always land on the same square numbers — this shows how different mathematical patterns can be deeply connected, and pictures make the connection visible and provable.

Q4. What is the sum of 1 + 2 + 3 + ... + 99 + 100 + 99 + ... + 3 + 2 + 1? Explain how you arrived at your answer using the pattern discussed in the chapter.

Answer:

- From the chapter, we know that adding counting numbers up to n and then back down to 1 always gives n^2 (e.g., $1+2+1 = 2^2$, $1+2+3+2+1 = 3^2$, and so on).
- In this case, the numbers go up to 100 and then back down, so n = 100.
- Therefore, the sum = 100^2 = 10,000.
- This can be visualised as a large diamond-shaped arrangement of dots with 100 dots in the widest (middle) row, tapering symmetrically to 1 dot at the top and bottom — which rearranges perfectly into a 100 × 100 square grid.

Q5. Explain how the sequence of hexagonal numbers (1, 7, 19, 37, ...) is related to (a) triangular numbers and (b) cube numbers.

Answer:

- (a) Relation to triangular numbers: Multiplying each triangular number by 6 and adding 1 gives the hexagonal numbers: (1×6)+1 = 7, (3×6)+1 = 19, (6×6)+1 = 37, (10×6)+1 = 61. This works because a hexagonal arrangement of dots can be split into 6 triangular sections around a central dot.
- (b) Relation to cube numbers: Adding up the hexagonal numbers in sequence gives the cube numbers: 1 = 1, 1+7 = 8, 1+7+19 = 27, 1+7+19+37 = 64. This can be explained using a picture of a cube — each hexagonal 'layer' of dots corresponds to a cross-sectional layer when building up a 3-D cube, so stacking these hexagonal layers together builds the full cube.

E. Homework — Practice Questions

Attempt the following in your notebook. These are for practice and self-assessment; hints are given where useful.

Section 1: Quick Recall

- 1. Write the first 8 terms of the sequence of cubes.
- 2. Write the rule for forming the Virahānka (Fibonacci) sequence and write its first 10 terms.
- 3. List the names of regular polygons having 3 to 10 sides, in order.
- 4. What is the sum of the first 20 odd numbers? (Hint: use n^2)
- 5. Find the next three numbers in the hexagonal number sequence 1, 7, 19, 37, ...

Section 2: Draw and Explain

- 1. Draw dot pictures to show that 36 is both a triangular number and a square number.
- 2. Draw the next picture (5th figure) for the Stacked Squares and Stacked Triangles sequences from Table 3.
- 3. Draw a pictorial explanation for why $1 + 2 + 3 + 2 + 1 = 3^2$ using a diamond of dots.
- 4. Draw K2, K3, K4, K5, K6 (Complete Graphs) and count the lines in each to verify the triangular number pattern.

Section 3: Apply and Reason

- 1. If you multiply each triangular number by 6 and add 1, which sequence do you get? Verify for the first 5 terms.
- 2. Find the sum of the first 6 hexagonal numbers and check whether it matches the corresponding cube number.
- 3. A Koch Snowflake starts with 3 segments. How many segments will it have after 4 steps of the pattern? Show your working.
- 4. Explain in your own words why the number of sides equals the number of corners in any regular polygon.
- 5. Create one new pattern of your own (numbers or shapes) and explain the rule that generates it.

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